1-4 Solving Absolute Value Equations

Evaluate each expression if $x = -4$ and $y = -9$.

1. $|x-8|$

ANSWER:
12

2. $|7y|$

ANSWER:
63

3. $-3|xy|$

ANSWER:
−108

4. $-2|3x+8|-4$

ANSWER:
−12

5. **MODELING** Most freshwater tropical fish thrive if the water is within 2°F of 78°F.
a. Write an equation to determine the least and greatest optimal temperatures.
b. Solve the equation you wrote in part a.
c. If your aquarium's thermometer is accurate to within plus or minus 1°F, what should the temperature of the water be to ensure that it reaches the minimum temperature? Explain.

ANSWER:
a. $|x-78|=2$
b. least: 76°F, greatest: 80°F
c. 77°F; This would ensure a minimum temperature of 76°F.

Solve each equation. Check your solutions.

6. $|x+8|=12$

ANSWER:
{4, –20}

7. $|y-4|=11$

ANSWER:
{15, –7}

8. $|a-5|+4=9$

ANSWER:
{10, 0}

9. $|b-3|+8=3$

ANSWER:
∅

10. $3|2x-3|-5=4$

ANSWER:
{3, 0}

11. $-2|5y-1|=-10$

ANSWER:
$\left\{\frac{6}{5},-\frac{4}{5}\right\}$

12. $|a-4|=3a-6$

ANSWER:
{2.5}

13. $|b+5|=2b+3$

ANSWER:
{2}

Evaluate each expression if $a = -3$, $b = -5$, and $c = 4.2$.

14. $|-3c|$

ANSWER:
12.6

15. $|5b|$

ANSWER:
25

16. $|a-b|$

ANSWER:
2

17. $|b-c|$

ANSWER:
9.2

18. $|3b-4a|$

ANSWER:
3

19. $2|4a-3c|$

ANSWER:
49.2

20. $-|3c-a|$

ANSWER:
–15.6

21. $-|abc|$

ANSWER:
–63

22. **FOOD** To make cocoa powder, cocoa beans are roasted. The ideal temperature for roasting is 300°F, plus or minus 25°. Write and solve an equation describing the maximum and minimum roasting temperatures for cocoa beans.

ANSWER:
$|x-300|=25$; maximum: 325°F; minimum: 275°F

Solve each equation. Check your solutions.

23. $|z-13|=21$

ANSWER:
{34, –8}

24. $|w+9|=17$

ANSWER:
{8, –26}

25. $9=|d+5|$

ANSWER:
{4, –14}

26. $35=|x-6|$

ANSWER:
{–29, 41}

27. $5|q+6|=20$

ANSWER:
{–2, –10}

28. $-3|r+4|=-21$

ANSWER:
{3, –11}

29. $3|2a-4|=0$

ANSWER:
{2}

30. $8|5w-1|=0$

ANSWER:
$\left\{\frac{1}{5}\right\}$

31. $2|3x-4|+8=6$

ANSWER:
$\varnothing$

32. $4|7y+2|-8=-7$

ANSWER:
$\left\{-\frac{1}{4},-\frac{9}{28}\right\}$

33. $-3|3t-2|-12=-6$

ANSWER:
$\varnothing$

34. $-5|3z+8|-5=-20$

ANSWER:

$\left\{-\frac{5}{3},-\frac{11}{3}\right\}$

35. **MONEY** The U.S. Mint produces quarters that weigh about 5.67 grams each. After the quarters are produced, a machine weighs them. If the quarter weighs 0.02 gram more or less than the desired weight, the quarter is rejected. Write and solve an equation to find the heaviest and lightest quarters the machine will approve.

ANSWER:

$|x-5.67|=0.02$; heaviest: 5.69 g; lightest: 5.65 g

Evaluate each expression if $q = -8$, $r = -6$, and $t = 3$.

36. $12-t|3r+2|$

ANSWER:

–36

37. $2q+|2rt+q|$

ANSWER:

28

38. $-5t-q|8r-t|$

ANSWER:

393

Solve each equation. Check your solutions.

39. $8x=2|6x-2|$

ANSWER:

$\left\{1,\frac{1}{5}\right\}$

40. $-6y+4=|4y+12|$

ANSWER:

$\left\{-\frac{4}{5}\right\}$

41. $8z+20=-|2z+4|$

ANSWER:

$\left\{-\frac{8}{3}\right\}$

42. $-3y-2=|6y+25|$

ANSWER:

$\left\{-3,-\frac{23}{3}\right\}$

43. **SEA LEVEL** Florida is on average 100 feet above sea level. This level varies by as much as 245 feet depending on precipitation and your location. Write and solve an equation describing the maximum and minimum sea levels for Florida. Is this solution reasonable? Explain.

ANSWER:

$|x-100|=245$; maximum: 345 ft above sea level; minimum: –145 ft below sea level. No, the maximum is reasonable but the minimum is not. Florida's lowest point should be at sea level where Florida meets the Atlantic Ocean and the Gulf of Mexico.

44. **MULTIPLE REPRESENTATIONS** Draw a number line.

a. GEOMETRIC Label any 5 integers on the number line points *A*, *B*, *C*, *D*, and *F*.

b. TABULAR Fill in each blank in the table with either > or < using the points from the number line.

$A \_\_ B$	$A+C \_\_ B+C$ $A+D \_\_ B+D$ $A+F \_\_ B+F$
$B \_\_ A$	$B+C \_\_ A+C$ $B+D \_\_ A+D$ $B+F \_\_ A+F$

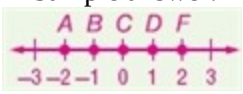
$A \_\_ B$	$A-C \_\_ B-C$ $A-D \_\_ B-D$ $A-F \_\_ B-F$
$B \_\_ A$	$B-C \_\_ A-C$ $B-D \_\_ A-D$ $B-F \_\_ A-F$

c. VERBAL Describe the patterns in the table.

d. ALGEBRAIC Describe the patterns algebraically, using the variable *x* to replace *C*, *D*, and *F*.

ANSWER:

a. Sample answer:



b.

$A \_\_ B$	$A+C \le B+C$ $A+D \le B+D$ $A+F \le B+F$
$B \_\_ A$	$B+C \ge A+C$ $B+D \ge A+D$ $B+F \ge A+F$

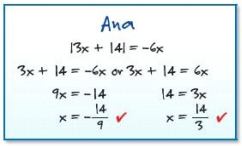
$A \_\_ B$	$A-C \le B-C$ $A-D \le B-D$ $A-F < B-F$
$B \_\_ A$	$B-C \ge A-C$ $B-D \ge A-D$ $B-F \ge A-F$

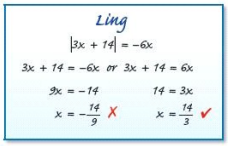
c. Sample answer: If *A* is less than *B*, then any number added to or subtracted from *A* will be less than the same number added to or subtracted from *B*. If *B* is greater than *A*, then any number added to or subtracted from *B* is greater than the same number added to or subtracted from *A*.

d. If $A < B$, then $A + x < B + x$. If $A < B$, then $A - x < B - x$.

If $B > A$, then $B + x > A + x$. If $B > A$, then $B - x > A - x$.

45. **CRITIQUE** Ana and Ling are solving $|3x + 14| = -6x$. Is either of them correct? Explain your reasoning.





ANSWER:

Ling; Ana included an extraneous solution. She would have caught this error if she had checked to see if her answers were correct by substituting the values into the original equation.

46. **CHALLENGE** Solve $|2x - 1| + 3 = |5 - x|$. List all cases and resulting equations. (*Hint*: There are four possible cases to examine as potential solutions.)

ANSWER:

The 4 potential solutions are:
1. $(2x - 1) \ge 0$ and $(5 - x) \ge 0$
2. $(2x - 1) \ge 0$ and $(5 - x) < 0$
3. $(2x - 1) < 0$ and $(5 - x) \ge 0$
4. $(2x - 1) < 0$ and $(5 - x) < 0$

The resulting equations corresponding to these cases are:
1. $2x - 1 + 3 = 5 - x : x = 1$
2. $2x - 1 + 3 = x - 5 : x = -7$
3. $1 - 2x + 3 = 5 - x : x = -1$
4. $1 - 2x + 3 = x - 5 : x = 3$

The solutions from case 1 and case 3 work. The others are extraneous. The solution set is {–1, 1}.

REASONING If *a*, *x*, and *y* are real numbers, determine whether each statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

47. If $|a| > 7$, then $|a + 3| > 10$.

ANSWER:

Sometimes; this is only true for certain values of *a*. For example, it is true for $a = 8$; if $8 > 7$, then $11 > 10$. However it is not true for $a = -8$; if $8 > 7$, then $5 \not> 10$.

48. If $|x| < 3$, then $|x| + 3 > 0$.

ANSWER:

Always; if $|x| < 3$, then x is between –3 or 3. Adding 3 to the absolute value of any of the numbers in this set will produce a positive number.

49. If y is between 1 and 5, then $|y - 3| \leq 2$.

ANSWER:

Always; starting with numbers between 1 and 5 and subtracting 3 will produce numbers between –2 and 2. These all have an absolute value less than or equal to 2.

50. **OPEN ENDED** Write an absolute value equation of the form $|ax + b| = cx + d$ that has no solution. Assume that a, b, c, and $d \neq 0$.

ANSWER:

Sample answer: $|2x + 1| = x - 3$, or $|3x + 10| = x - 5$, or $|x - 1| = \frac{1}{2}x - 4$

51. **WRITING IN MATH** How are symbols used to represent mathematical ideas? Use an example to justify your reasoning.

ANSWER:

Sample answer: Symbols can be used as a shorthand way to represent ideas such as operations, equality, absolute value, and the empty set. For example, instead of writing 5 minus the absolute value of $2x$ equals 10, you could write $5 - |2x| = 10$.

52. If $4x - y = 3$ and $2x + 3y = 19$, what is the value of y?

A 2
B 3
C 4
D 5

ANSWER:

D

53. **GRIDDED RESPONSE** Two male and 2 female students from each of the 9th, 10th, 11th, and 12th grades comprise the Student Council. If a Student Council representative is chosen at random to attend a board meeting, what is the probability that the student will be either an 11th grader or male?

ANSWER:

5/8

54. Which equation is equivalent to $4(9 - 3x) = 7 - 2(6 - 5x)$?

F $8x = 41$
G $22x = 41$
H $8x = 24$
J $22x = 24$

ANSWER:

G

55. **SAT/ACT** A square with side length 4 units has one vertex at the point (1, 2). Which one of the following points cannot be diagonally opposite that vertex?

A (–3, –2)
B (–3, 6)
C (5, –2)
D (5, 6)
E (1, 6)

ANSWER:

E

Solve each equation. Check your solution.

56. $4x + 6 = 30$

ANSWER:

6

57. $5p - 10 = 4(7 + 6p)$

ANSWER:

–2

58. $\frac{3}{5}y - 7 = \frac{2}{5}y + 3$

ANSWER:

50

59. **MONEY** Nhu is saving to buy a car. In the first 6 months, his savings were \$80 less than $\frac{3}{4}$ the price of the car. In the second six months, Nhu saved \$50 more than $\frac{1}{5}$ the price of the car. He still needs \$370.
a. What is the price of the car?
b. What is the average amount of money Nhu saved each month?
c. If Nhu continues to save the average amount each month, in how many months will he be able to afford the car?

ANSWER:
a. \$6800
b. \$535.83
c. 1 mo

Name the property illustrated by each equation.

60. $(1 + 8) + 11 = 11 + (1 + 8)$

ANSWER:
Comm. (+)

61. $z(9-4) = z \cdot 9 - z \cdot 4$

ANSWER:
Distributive

Simplify each expression.

62. $7a + 3b - 4a - 5b$

ANSWER:
$3a - 2b$

63. $3x + 5y + 7x - 3y$

ANSWER:
$10x + 2y$

64. $3(15x - 9y) + 5(4y - x)$

ANSWER:
$40x - 7y$

65. $2(10m - 7a) + 3(8a - 3m)$

ANSWER:
$11m + 10a$

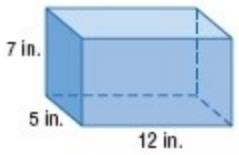
66. $8(r + 7t) - 4(13t + 5r)$

ANSWER:
$-12r + 4t$

67. $4(14c - 10d) - 6(d + 4c)$

ANSWER:
$32c - 46d$

68. **GEOMETRY** The formula for the surface area of a rectangular prism is $SA = 2\ell w + 2\ell h + 2wh$, where ℓ represents the length, w represents the width, and h represents the height. Find the surface area of the rectangular prism at the right.



ANSWER:
358 in^2

Solve each equation.

69. $15x + 5 = 35$

ANSWER:
2

70. $2.4y + 4.6 = 20$

ANSWER:
≈ 6.417

71. $8a + 9 = 6a - 7$

ANSWER:
-8

72. $3(w - 1) = 2w - 6$

ANSWER:
-3

73. $\frac{1}{2}(2b - 4) = 2 + 8b$

ANSWER:
$-\frac{4}{7}$

74. $\frac{1}{3}(6p-24)=18+3p$

ANSWER:
–26