Write a quadratic equation in standard form with the given root(s).

1. –8, 5

**SOLUTION:**
Write the pattern.

\[(x - p)(x - q) = 0\]

Replace \(p\) and \(q\) with –8 and 5.

\[(x - (-8))(x - 5) = 0\]
\[(x + 8)(x - 5) = 0\]

Use the FOIL method to multiply.

\[x(x) + x(-5) + 8(x) + 8(-5) = 0\]
\[x^2 - 5x + 8x - 40 = 0\]
\[x^2 + 3x - 40 = 0\]

**ANSWER:**
\[x^2 + 3x - 40 = 0\]

2. \(\frac{3}{2}, \frac{1}{4}\)

**SOLUTION:**
Write the pattern.

\[(x - p)(x - q) = 0\]

Replace \(p\) and \(q\) with \(\frac{3}{2}\) and \(\frac{1}{4}\).

\[\left(x - \frac{3}{2}\right)\left(x - \frac{1}{4}\right) = 0\]

Use the FOIL method to multiply.

\[x(x) + x\left(-\frac{1}{4}\right) - \frac{3}{2}(x) - \frac{3}{2}\left(-\frac{1}{4}\right) = 0\]
\[x^2 - \frac{1}{4}x - \frac{3}{2}x + \frac{3}{8} = 0\]

Multiply each side by 8.

\[8x^2 - 2x - 12x + 3 = 0\]
\[8x^2 - 14x + 3 = 0\]

**ANSWER:**
\[8x^2 - 14x + 3 = 0\]
4-3 Solving Quadratic Equations by Factoring

3. \( \frac{2}{3}, \frac{5}{2} \)

**SOLUTION:**
Write the pattern.

\[(x - p)(x - q) = 0\]

Replace \( p \) and \( q \) with \( -\frac{2}{3} \) and \( \frac{5}{2} \).

\[
\left(x - \left(-\frac{2}{3}\right)\right)\left(x - \frac{5}{2}\right) = 0
\]

\[
\left(x + \frac{2}{3}\right)\left(x - \frac{5}{2}\right) = 0
\]

Use the FOIL method to multiply.

\[
x(x) + x\left(-\frac{5}{2}\right) + \frac{2}{3}(x) + \frac{2}{3}\left(-\frac{5}{2}\right) = 0
\]

\[
x^2 - \frac{5}{2}x + \frac{2}{3}x - \frac{5}{3} = 0
\]

Multiply each side by 6.

\[
6x^2 - 15x + 4x - 10 = 0
\]

\[
6x^2 - 11x - 10 = 0
\]

**ANSWER:**

\( 6x^2 - 11x - 10 = 0 \)

---

Factor each polynomial.

4. \( 35x^2 - 15x \)

**SOLUTION:**
The GCF of the two terms is \( 5x \). Factor the GCF.

\[
35x^2 - 15x = 5x(7x) - 5x(3)
\]

\[
= 5x(7x - 3)
\]

**ANSWER:**

\( 5x(7x - 3) \)

5. \( 18x^2 - 3x + 24x - 4 \)

**SOLUTION:**
Factor \( 3x \) from the first two terms and 4 from the last two terms.

\[
18x^2 - 3x + 24x - 4
\]

\[
= 3x(6x - 1) + 4(6x - 1)
\]

Factor \( 6x - 1 \) from the two terms.

\[
3x(6x - 1) + 4(6x - 1) = (6x - 1)(3x + 4)
\]

Therefore,

\[
18x^2 - 3x + 24x - 4 = (6x - 1)(3x + 4).
\]

**ANSWER:**

\( (6x - 1)(3x + 4) \)
6. \( x^2 - 12x + 32 \)

**SOLUTION:**
Find the factors of 32 whose sum is \(-12\).

<table>
<thead>
<tr>
<th>Factors of 32</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,32</td>
<td>33</td>
</tr>
<tr>
<td>(-1,\overline{32})</td>
<td>(-33)</td>
</tr>
<tr>
<td>2,16</td>
<td>18</td>
</tr>
<tr>
<td>(-2,\overline{16})</td>
<td>(-18)</td>
</tr>
<tr>
<td>4,8</td>
<td>12</td>
</tr>
<tr>
<td>(-4,\overline{8})</td>
<td>(-12)</td>
</tr>
</tbody>
</table>

Write \(-12x\) as \((-4)x + (-8)x\).

\[ x^2 - 12x + 32 = x^2 - 4x - 8x + 32 \]

Factor \( x \) from the first two terms and \(-8\) from the last two terms.

\[ x^2 - 4x - 8x + 32 = x(x - 4) - 8(x - 4) \]

Factor \( x - 4 \) from the two terms.

\[ x(x - 4) - 8(x - 4) = (x - 4)(x - 8) \]

Therefore,

\[ x^2 - 12x + 32 = (x - 4)(x - 8). \]

**ANSWER:**
\((x - 8)(x - 4)\)

7. \( x^2 - 4x - 21 \)

**SOLUTION:**
Find the factors of \(-21\) whose sum is \(-4\).

<table>
<thead>
<tr>
<th>Factors of (-21)</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,(-21)</td>
<td>(-20)</td>
</tr>
<tr>
<td>(-1,21)</td>
<td>20</td>
</tr>
<tr>
<td>3,(-7)</td>
<td>(-4)</td>
</tr>
<tr>
<td>(-3,7)</td>
<td>4</td>
</tr>
</tbody>
</table>

Write \(-4x\) as \((-7)x + 3x\).

\[ x^2 - 4x - 21 = x^2 - 7x + 3x - 21 \]

Factor \( x \) from the first two terms and 3 from the last two terms.

\[ x^2 - 7x + 3x - 21 = x(x - 7) + 3(x - 7) \]

Factor \( x - 7 \) from the two terms.

\[ x(x - 7) + 3(x - 7) = (x - 7)(x + 3) \]

Therefore,

\[ x^2 - 4x - 21 = (x - 7)(x + 3). \]

**ANSWER:**
\((x - 7)(x + 3)\)
8. \(2x^2 + 7x - 30\)

**SOLUTION:**
Here, \(a = 2\), \(b = 7\) and \(c = -30\).

\[ac = 2(-30) = -60\]

Find two factors of \(-60\) whose sum is \(7\).

\(12(-5) = -60\) and \(12 + (-5) = 7\)

Write \(7x\) as \(12x + (-5x)\).

\[2x^2 + 7x - 30 = 2x^2 + 12x - 5x - 30\]

Factor \(2x\) from the first two terms and \(-5\) from the last two terms.

\[2x^2 + 12x - 5x - 30 = 2x(x + 6) - 5(x + 6)\]

Factor \(x + 6\) from the two terms.

\[2x(x + 6) - 5(x + 6) = (x + 6)(2x - 5)\]

Therefore,

\[2x^2 + 7x - 30 = (2x - 5)(x + 6)\]

**ANSWER:**
\((2x - 5)(x + 6)\)

9. \(16x^2 - 16x + 3\)

**SOLUTION:**
Here, \(a = 16\), \(b = -16\) and \(c = 3\).

\[ac = 16(3) = 48\]

Find two factors of \(48\) whose sum is \(-16\)

\(-12(-4) = 48\) and \(-12 + (-5) = -16\)

Write \(-16x\) as \(-12x + (-4x)\).

\[16x^2 - 16x + 3 = 16x^2 - 12x - 4x + 3\]

Factor \(4x\) from the first two terms and \(-1\) from the last two terms.

\[16x^2 - 12x - 4x + 3 = 4x(4x - 3) - 1(4x - 3)\]

Factor \(4x - 3\) from the two terms.

\[4x(4x - 3) - 1(4x - 3) = (4x - 3)(4x - 1)\]

Therefore,

\[16x^2 - 16x + 3 = (4x - 3)(4x - 1)\]

**ANSWER:**
\((4x - 3)(4x - 1)\)
4-3 Solving Quadratic Equations by Factoring

Solve each equation.

10. \(x^2 - 36 = 0\)

SOLUTION:
Use the identity \(a^2 - b^2 = (a + b)(a - b)\).
Here, \(a = x\) and \(b = 6\).

\[x^2 - 36 = (x)^2 - (6)^2 = (x + 6)(x - 6)\]
\[x = -6, 6\]

ANSWER: 
-6, 6

11. \(12x^2 - 18x = 0\)

SOLUTION:
The GCF of the two terms is 6x. Factor the GCF.

\[12x^2 - 18x = 0\]
\[= 6x(2x) - 6x(3) = 0\]
\[= 6x(2x - 3) = 0\]
\[6x = 0 \text{ or } 2x - 3 = 0\]
\[x = 0 \text{ or } x = \frac{3}{2}\]

ANSWER: 
0, \(\frac{3}{2}\)

12. \(12x^2 - 2x - 2 = 0\)

SOLUTION:
Here, \(a = 12\), \(b = -2\) and \(c = -2\).

\[ac = 12(-2) = -24\]

Find two factors of \(-24\) whose sum is \(-2\).

\[-6(4) = -24\text{ and } -6 + 4 = -2\]

Write \(-2x\) as \(-6x + 4x\).

\[12x^2 - 2x - 2 = 12x^2 - 6x + 4x - 2\]

Factor \(6x\) from the first two terms and 2 from the last two terms.

\[12x^2 - 6x + 4x - 2 = 6x(2x - 1) + 2(2x - 1)\]

Factor \(2x - 1\) from the two terms.

\[6x(2x - 1) + 2(2x - 1) = (2x - 1)(6x + 2) = (2x - 1)(3x + 1)\]

Therefore,

\[12x^2 - 2x - 2 = 2(2x - 1)(3x + 1)\]

\[(2x - 1) = 0 \text{ or } (3x + 1) = 0\]

\[2x = 1 \text{ or } 3x = -1\]

\[x = \frac{1}{2} \text{ or } x = -\frac{1}{3}\]

ANSWER: 
\(-\frac{1}{3}, \frac{1}{2}\)
13. \( x^2 - 9x = 0 \)

**SOLUTION:**
The GCF of the two terms on the left is \( x \). Factor the GCF.

\[ x(x - 9) = 0 \]

Use the Zero Product Property.

\[ x(x - 9) = 0 \iff x = 0 \quad \text{or} \quad x - 9 = 0 \]

\[ \iff x = 0 \quad \text{or} \quad x = 9 \]

Therefore, the roots are 0 and 9.

**ANSWER:**
0, 9

14. \( x^2 - 3x - 28 = 0 \)

**SOLUTION:**
Find the factors of \(-28\) whose sum is \(-3\).

\[ 4(-7) = -28 \quad \text{and} \quad -7 + 4 = -3 \]

Write \(-3x\) as \(4x + (-7x)\).

\[ x^2 - 3x - 28 = 0 \]

\[ x^2 + 4x - 7x - 28 = 0 \]

Factor \( x\) from the first two terms and \(-7\) from the last two terms.

\[ x^2 + 4x - 7x - 28 = x(x + 4) - 7(x + 4) = 0 \]

Factor \( x + 4 \) from the two terms.

\[ x(x + 4) - 7(x + 4) = 0 \]

\[ (x + 4)(x - 7) = 0 \]

Use the Zero Product Property.

\[ (x + 4)(x - 7) = 0 \iff x + 4 = 0 \quad \text{or} \quad x - 7 = 0 \]

\[ \iff x = -4 \quad \text{or} \quad x = 7 \]

Therefore, the roots are \(-4\) and 7.

**ANSWER:**
\(-4, 7\)
15. \(2x^2 - 24x = -72\)

**SOLUTION:**
Divide each side of the equation by 2.

\[x^2 - 12x = -36\]

Write the equation with the right side equals zero.

\[x^2 - 12x + 36 = 0\]

Use the identity \((a - b)^2 = a^2 - 2ab + b^2\) to factor the left side of the equation.

Here, \(a = x\) and \(b = 6\).

\[x^2 - 12x + 36 = (x)^2 - 2(x)(6) + (6)^2\]

\[= (x - 6)^2\]

So, \((x - 6)^2 = 0\).

Use the Zero Product Property.

\[(x - 6)^2 = 0 \Rightarrow x - 6 = 0\]

\[\Rightarrow x = 6\]

Therefore, the root is 6 and it is a repeated root.

**ANSWER:**
6

16. **CCSS SENSE-MAKING** Tamika wants to double the area of her garden by increasing the length and width by the same amount. What will be the dimensions of her garden then?

**SOLUTION:**

The area of a rectangle of length \(l\) and width \(w\) is \(lw\). So, the area of the garden is \(9(6) = 54\ m^2\).

Let \(x\) be the amount in length and width that has to be increased to double the area. Then,

\[(9 + x)(6 + x) = 2(54)\]

\[(9 + x)(6 + x) = 108\]

Use the FOIL method to multiply the left.

\[x(x) + x(6) + 9(x) + 9(6) = 108\]

\[x^2 + 6x + 9x + 54 = 108\]

\[x^2 + 15x - 54 = 0\]

Find the factors of \(-54\) whose sum is 15.

\[18(-3) = -54\text{ and }-3 + 18 = 15\]

Write \(15x\) as \(-3x + 18x\).

\[x^2 + 15x - 54 = 0\]

\[x^2 - 3x + 18x - 54 = 0\]

Factor \(x\) from the first two terms and 18 from the last two terms.

\[x^2 - 3x + 18x - 54 = x(x - 3) + 18(x - 3) = 0\]

Factor \(x - 3\) from the two terms.

\[x(x - 3) + 18(x - 3) = (x - 3)(x + 18) = 0\]

\[x^2 + 15x - 54 = (x - 3)(x + 18) = 0\]

Use the Zero Product Property.

\[(x - 3)(x + 18) = 0 \Rightarrow x - 3 = 0 \text{ or } x + 18 = 0\]

\[\Rightarrow x = 3 \text{ or } x = -18\]

So, the roots are 3 and \(-18\).

But \(x\) is a length, so it cannot be negative.

So, \(x = 3\).

Therefore, 3 m should be added to the length and width to double the area. The new dimensions of
the garden will be 9 m by 12 m.

**ANSWER:**
9 m by 12 m

Write a quadratic equation in standard form with the given root(s).

17. 7

**SOLUTION:**
Write the pattern.

\[(x - p)(x - q) = 0\]

Since there is only one root, it is a repeated root.
Replace \(p\) and \(q\) with 7.

\[(x - 7)(x - 7) = 0\]

Use the FOIL method to multiply.

\[x(x) + x(-7) - 7(x) - 7(-7) = 0\]
\[x^2 - 7x - 7x + 49 = 0\]
\[x^2 - 14x + 49 = 0\]

**ANSWER:**
\[x^2 - 14x + 49 = 0\]

18. \(-5, \frac{1}{2}\)

**SOLUTION:**
Write the pattern.

\[(x - p)(x - q) = 0\]

Replace \(p\) and \(q\) with \(-5\) and \(\frac{1}{2}\).

\[(x - (-5))(x - \frac{1}{2}) = 0\]
\[(x + 5)(x - \frac{1}{2}) = 0\]

Use the FOIL method to multiply.

\[x(x) + x\left(\frac{1}{2}\right) + 5(x) + 5\left(-\frac{1}{2}\right) = 0\]
\[x^2 + \frac{1}{2}x + 5x - \frac{5}{2} = 0\]

Multiply each side by 2.

\[2x^2 - x + 10x - 5 = 0\]
\[2x^2 + 9x - 5 = 0\]

**ANSWER:**
\[2x^2 + 9x - 5 = 0\]
4-3 Solving Quadratic Equations by Factoring

19. \( \frac{1}{5}, 6 \)

**SOLUTION:**
Write the pattern.

\[(x - p)(x - q) = 0\]

Replace \( p \) and \( q \) with \( \frac{1}{5} \) and 6.

\[\left(x - \frac{1}{5}\right)(x - 6) = 0\]

Use the FOIL method to multiply.

\[x(x) + x(-6) - \frac{1}{5}(x) - \frac{1}{5}(-6) = 0\]

\[x^2 - 6x - \frac{1}{5}x + \frac{6}{5} = 0\]

Multiply each side by 5.

\[5x^2 - 30x - x + 6 = 0\]
\[5x^2 - 31x + 6 = 0\]

**ANSWER:**
\[5x^2 - 31x + 6 = 0\]

**Factor each polynomial.**

20. \( 40a^2 - 32a \)

**SOLUTION:**
The GCF of the two terms is \( 8a \). Factor the GCF.

\[40a^2 - 32a = 8a(5a) - 8a(4)\]

\[= 8a(5a - 4)\]

**ANSWER:**
\[8a(5a - 4)\]

21. \( 51c^3 - 34c \)

**SOLUTION:**
The GCF of the two terms is \( 17c \). Factor the GCF.

\[51c^3 - 34c = 17c(3c^2) - 17c(2)\]

\[= 17c(3c^2 - 2)\]

**ANSWER:**
\[17c(3c^2 - 2)\]

22. \( 32xy + 40bx - 12ay - 15ab \)

**SOLUTION:**
Factor \( 8x \) from the first two terms and \( -3a \) from the last two terms.

\[32xy + 40bx - 12ay - 15ab\]

\[= 8x(4y + 5b) - 3a(4y + 5b)\]

Factor \( 4y + 5b \) from the two terms.

\[8x(4y + 5b) - 3a(4y + 5b)\]

\[= (4y + 5b)(8x - 3a)\]

Therefore,

\[32xy + 40bx - 12ay - 15ab\]

\[= (4y + 5b)(8x - 3a)\]

**ANSWER:**
\[(8x - 3a)(4y + 5b)\]
4.3 Solving Quadratic Equations by Factoring

23. $3x^2 - 12$

**SOLUTION:**
Factor out 3.

$$3x^2 - 12 = 3(x^2 - 4)$$

Use the identity $a^2 - b^2 = (a + b)(a - b)$ to factor $x^2 - 4$.

$$x^2 - 4 = (x + 2)(x - 2)$$

Therefore,

$$3x^2 - 12 = 3(x + 2)(x - 2).$$

**ANSWER:**
$3(x + 2)(x - 2)$

24. $15y^2 - 240$

**SOLUTION:**
Factor out 15.

$$15y^2 - 240 = 15(y^2 - 16)$$

Use the identity $a^2 - b^2 = (a + b)(a - b)$ to factor $y^2 - 16$.

$$y^2 - 16 = (y + 4)(y - 4)$$

Therefore,

$$15y^2 - 240 = 15(y + 4)(y - 4).$$

**ANSWER:**
$15(y + 4)(y - 4)$

25. $48cg + 36cf - 4dg - 3df$

**SOLUTION:**
Factor 12$c$ from the first two terms and $-d$ from the last two terms.

$$48cg + 36cf - 4dg - 3df = 12c(4g + 3f) - d(4g + 3f)$$

Factor $4g + 3f$ from the two terms.

$$12c(4g + 3f) - d(4g + 3f) = (4g + 3f)(12c - d)$$

Therefore,

$$48cg + 36cf - 4dg - 3df = (4g + 3f)(12c - d).$$

**ANSWER:**
$(12c - d)(4g + 3f)$
26. \(x^2 + 13x + 40\)

**SOLUTION:**
Find the factors of 40 whose sum is 13.

\[5(8) = 40 \text{ and } 5 + 8 = 13\]

Write \(13x\) as \(5x + 8x\).

\[x^2 + 13x + 40 = x^2 + 5x + 8x + 40\]

Factor \(x\) from the first two terms and 8 from the last two terms.

\[x^2 + 5x + 8x + 40 = x(x + 5) + 8(x + 5)\]

Factor \(x + 5\) from the two terms.

\[x(x + 5) + 8(x + 5) = (x + 5)(x + 8)\]

Therefore,

\[x^2 + 13x + 40 = (x + 5)(x + 8)\]

**ANSWER:**
\((x + 8)(x + 5)\)

27. \(x^2 - 9x - 22\)

**SOLUTION:**
Find the factors of \(-22\) whose sum is \(-9\).

\[2(-11) = -22 \text{ and } 2 + (-11) = -9\]

Write \(-9x\) as \(2x - 11x\).

\[x^2 - 9x - 22 = x^2 + 2x - 11x - 22\]

Factor \(x\) from the first two terms and \(-11\) from the last two terms.

\[x^2 + 2x - 11x - 22 = x(x + 2) - 11(x + 2)\]

Factor \(x + 2\) from the two terms.

\[x(x + 2) - 11(x + 2) = (x + 2)(x - 11)\]

Therefore,

\[x^2 - 9x - 22 = (x + 2)(x - 11)\]

**ANSWER:**
\((x - 11)(x + 2)\)
28. $3x^2 + 12x - 36$

**SOLUTION:**

Here, $a = 3$, $b = 12$ and $c = -36$.

$ac = 3(-36) = -108$

Find two factors of $-108$ whose sum is 12.

$-6(18) = -108$ and $-6 + 18 = 12$

Write $12x$ as $-6x + 18x$.

$3x^2 + 12x - 36 = 3x^2 - 6x + 18x - 36$

Factor $3x$ from the first two terms and 18 from the last two terms.

$3x^2 - 6x + 18x - 36 = 3x(x - 2) + 18(x - 2)$

Factor $x - 2$ from the two terms.

$3x(x - 2) + 18(x - 2) = (x - 2)(3x + 18)$

$= (x - 2)(3)(x + 6)$

$= 3(x - 2)(x + 6)$

Therefore,

$3x^2 + 12x - 36 = 3(x - 2)(x + 6)$.

**ANSWER:**

$3(x + 6)(x - 2)$

29. $15x^2 + 7x - 2$

**SOLUTION:**

Here, $a = 15$, $b = 7$ and $c = -2$.

$ac = 15(-2) = -30$

Find two factors of $-30$ whose sum is 7.

$10(-3) = -30$ and $10 + (-3) = 7$

Write $7x$ as $10x - 3x$.

$15x^2 + 7x - 2 = 15x^2 + 10x - 3x - 2$

Factor $5x$ from the first two terms and $-1$ from the last two terms.

$15x^2 + 10x - 3x - 2 = 5x(3x + 2) - 1(3x + 2)$

Factor $3x + 2$ from the two terms.

$5x(3x + 2) - 1(3x + 2) = (3x + 2)(5x - 1)$

Therefore,

$15x^2 + 7x - 2 = (3x + 2)(5x - 1)$.

**ANSWER:**

$(5x - 1)(3x + 2)$
Write a quadratic equation in standard form with the given root(s).

1. –8, 5

**SOLUTION:**
Here, \( a = 4, b = 29 \) and \( c = 30 \).

\[ ac = 4(30) = 120 \]

Find two factors of 120 whose sum is 29.

\[ 5(24) = 120 \text{ and } 5 + 24 = 29 \]

Write 29\(x\) as 5\(x\) + 24\(x\).

\[ 4x^2 + 29x + 30 = 4x^2 + 5x + 24x + 30 \]

Factor \(x\) from the first two terms and 6 from the last two terms.

\[ 4x^2 + 5x + 24x + 30 = x(4x + 5) + 6(4x + 5) \]

Factor \(4x + 5\) from the two terms.

\[ x(4x + 5) + 6(4x + 5) = (4x + 5)(x + 6) \]

Therefore,

\[ 4x^2 + 29x + 30 = (4x + 5)(x + 6). \]

**ANSWER:**

\((4x + 5)(x + 6)\)

---

30. \(4x^2 + 29x + 30\)

31. \(18x^2 + 15x - 12\)

**SOLUTION:**
Here, \( a = 18, b = 15 \) and \( c = -12 \).

\[ ac = 18(-12) = -216 \]

Find two factors of –216 whose sum is 15.

\[ 24(-9) = -216 \text{ and } 24 + (-9) = 15 \]

Write 15\(x\) as 24\(x\) + (–9)\(x\).

\[ 18x^2 + 15x - 12 = 18x^2 + 24x - 9x - 12 \]

Factor 6\(x\) from the first two terms and –3 from the last two terms.

\[ 18x^2 + 24x - 9x - 12 = 6x(3x + 4) - 3(3x + 4) \]

Factor 3\(x + 4\) from the two terms.

\[ 6x(3x + 4) - 3(3x + 4) = (6x - 3)(3x + 4) \]

Therefore,

\[ 18x^2 + 15x - 12 = 3(2x - 1)(3x + 4). \]

**ANSWER:**

\(3(2x - 1)(3x + 4)\)

32. \(8x^2z^2 - 4xz^2 - 12z^2\)

**SOLUTION:**
Factor \(z^2\) from all the three terms.

\[ 8x^2z^2 - 4xz^2 - 12z^2 \]

\[ = z^2 \left( 8x^2 - 4x - 12 \right) \]

Factor \(8x^2 - 4x - 12\).

Here, \( a = 8, b = -4 \) and \( c = -12 \).
4-3 Solving Quadratic Equations by Factoring

\[ ac = 8(-12) = -96 \]

Find two factors of \(-96\) whose sum is \(-4\).

\(-12(8) = -96\) and \(-12 + 8 = -4\)

Write \(-4x\) as \(-12x + 8x\).

\[ 8x^2 - 4x - 12 \]

\[ = 8x^2 - 12x + 8x - 12 \]

Factor \(4x\) from the first two terms and \(4\) from the last two terms.

\[ 8x^2 - 12x + 8x - 12 \]

\[ = 4x(2x - 3) + 4(2x - 3) \]

Factor \(2x - 3\) from the two terms.

\[ 4x(2x - 3) + 4(2x - 3) \]

\[ = (2x - 3)(4x + 4) \]

\[ = (2x - 3)(4)(x + 1) \]

\[ = 4(x + 1)(2x - 3) \]

Therefore,

\[ 8x^2z^2 - 4xz^2 - 12z^2 \]

\[ = 4z^2(x + 1)(2x - 3) \cdot \]

\textbf{ANSWER:}

\[ 4z^2(2x - 3)(x + 1) \]

\[ 33. \quad 9x^2 - 25 \]

\textbf{SOLUTION:}

Use the identity \(a^2 - b^2 = (a + b)(a - b)\)

\[ 9x^2 - 25 = (3x)^2 - (5)^2 \]

\[ = (3x + 5)(3x - 5) \]

Therefore,

\[ 9x^2 - 25 = (3x + 5)(3x - 5). \]

\textbf{ANSWER:}

\[ (3x + 5)(3x - 5) \]

\[ 34. \quad 18x^2y^2 - 24xy^2 + 36y^2 \]

\textbf{SOLUTION:}

The GCF of the three terms is \(6y^2\). Factor the GCF.

\[ 18x^2y^2 - 24xy^2 + 36y^2 \]

\[ = 6y^2(3x^2) - 6y^2(4x) + 6y^2(6) \]

\[ = 6y^2(3x^2 - 4x + 6) \]

\textbf{ANSWER:}

\[ 6y^2(3x^2 - 4x + 6) \]
Write a quadratic equation in standard form
with the given root(s).

1. –8, 5

SOLUTION:

Write the pattern.

Replace p and q with 5 and 1.

Use the FOIL method to multiply.

Multiply each side by 56.

Therefore, the roots are 

Find factors of 20(22) = 440 and 22 + 44 = 66.

Divide each side of the equation by 3.

ANSWER:

Find factors of 4(9) = 36 whose sum is 12.

Divide each side of the equation by 4.

Therefore, the roots are 0 and 3.

Solve each equation by factoring.

FT.

The area of a rectangle of length

GEOMETRY Find

When

Therefore, the roots are 8, 2.

Find two factors of 12(5) = 60 and 5 + 6 = 11.

Find two factors of 9(4) = 36 and 4 + 9 = 13.

Find the factors of 3(5) = 15 whose sum is 12.

Find two factors of 6(2) = 12 and 2 + 6 = 8.

Find the factors of 40 whose sum is 13.

Find two factors of 8(21) = 168 and 21 + 8 = 29.

Factor 3 from all the three terms.

Find two factors of –60 whose sum is –28.

Write –28x as –30x + 2x.

Factor 5x from the first two terms and 2 from the last two terms.

Factor x – 6 from the two terms.

Therefore,

\[15x^2 - 84x - 36 = 3(5x + 2)(x - 6)\]

ANSWER:

\[3(5x + 2)(x - 6)\]
4-3 Solving Quadratic Equations by Factoring

37. \(12xy^2 - 108x\)

**SOLUTION:**
Factor out the GCF, 12x.
\(12xy^2 - 108x = 12x(y^2 - 9)\)

Use the identity \(a^2 - b^2 = (a + b)(a - b)\) to factor \(y^2 - 9\).
\(y^2 - 9 = (y + 3)(y - 3)\)

Therefore,
\(12xy^2 - 108x = 12x(y + 3)(y - 3)\).

**ANSWER:**
\(12x(y + 3)(y - 3)\)

---

Solve each equation by factoring.

38. \(x^2 + 4x - 45 = 0\)

**SOLUTION:**
Find the factors of \(-45\) whose sum is 4.
\(9(-5) = -45 \text{ and } -5 + 9 = 4\)

Write \(4x\) as \(-5x + 9x\).
\(x^2 + 4x - 45 = x^2 - 5x + 9x - 45 = 0\)

Factor \(x\) from the first two terms and 9 from the last two terms.
\(x^2 + 5x - 9x - 45 = 0\)
\(x(x + 5) - 9(x + 5) = 0\)

Factor \(x + 5\) from the two terms.
\((x + 5)(x - 9) = 0\)

Use the Zero Product Property.
\((x + 5)(x - 9) = 0 \Rightarrow x + 5 = 0 \text{ or } x - 9 = 0\)
\(\Rightarrow x = -5 \text{ or } x = 9\)

Therefore,
the roots are \(-5\) and 9.

**ANSWER:**
\(5, -9\)
4-3 Solving Quadratic Equations by Factoring

39. \( x^2 - 5x - 24 = 0 \)

**SOLUTION:**
Find the factors of \(-24\) whose sum is \(-5\).

\[ 3(-8) = -24 \text{ and } 3 + (-8) = -5 \]

Write \(-5x\) as \(3x + (-5x)\).

\[ x^2 - 5x - 24 = 0 \]
\[ x^2 + 3x - 8x - 24 = 0 \]

Factor \(x\) from the first two terms and \(-8\) from the last two terms.

\[ x(x + 3) - 8(x + 3) = 0 \]
\[ (x + 3)(x - 8) = 0 \]

Factor \(x + 3\) from the two terms.

\[ x(x + 3) - 8(x + 3) = 0 \]
\[ (x + 3)(x - 8) = 0 \]

Use the Zero Product Property.

\[ (x + 3)(x - 8) = 0 \Rightarrow x + 3 = 0 \text{ or } x - 8 = 0 \]
\[ x = -3 \text{ or } x = 8 \]

Therefore, the roots are \(-3\) and \(8\).

**ANSWER:**
\(8, -3\)

40. \( x^2 = 121 \)

**SOLUTION:**
Write the equation with right side equal to zero.

\[ x^2 - 121 = 0 \]

Use the identity \(a^2 - b^2 = (a + b)(a - b)\) to factor.

\[ x^2 - 121 = 0 \]
\[ (x + 11)(x - 11) = 0 \]

Use the Zero Product Property.

\( (x + 11)(x - 11) = 0 \)
\[ x + 11 = 0 \text{ or } x - 11 = 0 \]
\[ x = -11 \text{ or } x = 11 \]

Therefore, the roots are \(-11\) and \(11\).

**ANSWER:**
\(11, -11\)
4-3 Solving Quadratic Equations by Factoring

41. \( x^2 + 13 = 17 \)

**SOLUTION:**
Write the equation with right side equal to zero.
\[
x^2 + 13 - 17 = 0
\]
\[
x^2 - 4 = 0
\]
Use the identity \( a^2 - b^2 = (a + b)(a - b) \) to factor \( x^2 - 4 \).
\[
x^2 - 4 = (x + 2)(x - 2) = 0
\]
Use the Zero Product Property.
\[(x + 2)(x - 2) = 0 \Rightarrow x + 2 = 0 \text{ or } x - 2 = 0 \]
\[\Rightarrow x = -2 \text{ or } x = 2\]
Therefore, the roots are -2 and 2.

**ANSWER:**
2, -2

42. \( -3x^2 - 10x + 8 = 0 \)

**SOLUTION:**
Factor out \(-1\).

\[
-1\left(3x^2 + 10x - 8\right) = 0
\]
\[
3x^2 + 10x - 8 = 0
\]
Now factor \(3x^2 + 10x - 8\).
Here, \(a = 3\), \(b = 10\) and \(c = -8\).
\[
ac = 3(-8) = -24
\]
Find two factors of \(-24\) whose sum is \(10\).
\[
12(-2) = -24 \text{ and } 12 + (-2) = 10
\]
Write \(10x\) as \(12x + (-2x)\).
\[
3x^2 + 10x - 8 = 3x^2 + 12x - 2x - 8
\]
Factor \(3x\) from the first two terms and \(-2\) from the last two terms.
\[
3x(x + 4) - 2(x + 4) = 0
\]
Factor \(x + 4\) from the two terms.
\[
3x(x + 4) - 2(x + 4) = 0
\]
Use the Zero Product Property.
\[
(x + 4)(3x - 2) = 0
\]
\[\Rightarrow x + 4 = 0 \text{ or } 3x - 2 = 0
\]
\[\Rightarrow x = -4 \text{ or } x = \frac{2}{3}\]
Therefore, the roots are \(-4\) and \(\frac{2}{3}\).

**ANSWER:**
\(-4, \frac{2}{3}\)

43. \( -8x^2 + 46x - 30 = 0 \)

**SOLUTION:**
Factor out \(-1\).

\[
-1\left(8x^2 - 46x + 30\right) = 0
\]
\[
8x^2 - 46x + 30 = 0
\]
Now factor \(8x^2 - 46x + 30\).
Here, \(a = 8\), \(b = -46\) and \(c = 30\).
\[
ac = 8(30) = 240
\]
Find two factors of \(240\) whose sum is \(-46\).
4-3 Solving Quadratic Equations by Factoring

\[-40(-6) = 240 \text{ and } -40 + (-6) = -46\]

Write \(-46x\) as \(-40x + (-6x)\).

\[8x^2 - 46x + 30 = 8x^2 - 40x - 6x + 30\]

Factor 8x from the first two terms and -6 from the last two terms.

\[8x^2 - 40x - 6x + 30 = 0\]

\[8x(x - 5) - 6(x - 5) = 0\]

Factor \(x - 5\) from the two terms.

\[(x - 5)(8x - 6) = 0\]

Use the Zero Product Property.

\[(x - 5)(8x - 6) = 0 \implies x - 5 = 0 \text{ or } 8x - 6 = 0\]

\[\implies x = 5 \text{ or } x = \frac{6}{8} = \frac{3}{4}\]

Therefore, the roots are 5 and \(\frac{3}{4}\).

**ANSWER:**

\[5, \frac{3}{4}\]

44. **GEOMETRY** The hypotenuse of a right triangle is 1 centimeter longer than one side and 4 centimeters longer than three times the other side. Find the dimensions of the triangle.

**SOLUTION:**

Let \(x\) be the length of the one of the legs. Then the length of the hypotenuse is \(3x + 4\) and that of the other leg is \(3x + 3\).

By the Pythagorean Theorem, the sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse.

\[(3x + 4)^2 = (3x + 3)^2 + x^2\]

Simplify and write in the standard form of a quadratic equation.

\[9x^2 + 24x + 16 = 9x^2 + 18x + 9 + x^2\]

\[x^2 - 6x - 7 = 0\]

Find two factors of -7 whose sum is -6.

\[1(-7) = -7 \text{ and } 1 + (-7) = -6\]

Write \(-6x\) as \(x + (-7x)\).

\[x^2 - 6x - 7 = 0\]

\[x^2 + x - 7x - 7 = 0\]

Factor \(x\) from the first two terms and -7 from the last two terms.

\[x(x + 1) - 7(x + 1) = 0\]

Factor \(x + 1\) from the two terms.

\[(x + 1)(x - 7) = 0\]

Use the Zero Product Property.

\[(x + 1)(x - 7) = 0 \implies x + 1 = 0 \text{ or } x - 7 = 0\]

\[\implies x = -1 \text{ or } x = 7\]

But \(x\) is a length; it cannot be negative. So, \(x = 7\).

Therefore, the lengths of the sides are 7 cm, 24 cm, and 25 cm.

**ANSWER:**

7 cm, 24 cm, 25 cm
45. **NUMBER THEORY** Find two consecutive even integers with a product of 624.

**SOLUTION:**
Let the numbers be $2n$ and $2(n + 1)$.

Their product is 624.

$2n(2(n + 1)) = 624$

$4n^2 + 4n - 624 = 0$

Here, $a = 4$, $b = 4$ and $c = 624$.

$ac = 4(624) = 2496$

Find two factors of 2496 whose sum is 4.

$52(-48) = 2496$ and $52 + (-48) = 4$

Write $4n$ as $52n - 48n$.

$4n^2 + 4n - 624 = 0$

$4n^2 + 52n - 48n - 624 = 0$

Factor $4n$ from the first two terms and $-48$ from the last two terms.

$4n^2 + 52n - 48n - 624 = 0$

$4n(n + 13) - 48(n + 13) = 0$

Factor $n + 13$ from the two terms.

$(4n - 48)(n + 13) = 0$

Use the Zero Product Property.

$(4n - 48)(n + 13) = 0 \Rightarrow 4n - 48 = 0 \quad \text{or} \quad n + 13 = 0$

$\Rightarrow n = 12 \quad \text{or} \quad n = -13$

When $n = 12$, the numbers are 24 and 26.

When $n = -13$, the numbers are $-24$ and $-26$.

**ANSWER:**
24 and 26 or -24 and -26

---

**GEOMETRY** Find $x$ and the dimensions of each rectangle.

**SOLUTION:**

The area of a rectangle of length $l$ and width $w$ is $l \times w$.

Here, $l = x + 2$, $w = x - 2$, and area = 96.

$(x + 2)(x - 2) = 96$

$x^2 - 4 = 96$

$x^2 = 100$

$x = \pm 10$

When $x = -10$, the dimensions of the rectangle becomes negative. So, $x = 10$.

The length of the rectangle is 12 ft and width is 8 ft.

**ANSWER:**

$x = 10$; 8 ft by 12 ft
4-3 Solving Quadratic Equations by Factoring

**SOLUTION:**
The area of a rectangle of length \( l \) and width \( w \) is \( l \times w \).

Here, \( l = x + 4 \), \( w = x - 2 \), and area = 432.

\[
\begin{align*}
(x + 4)(x - 2) &= 432 \\
x^2 - 2x + 4x - 8 &= 432 \\
x^2 + 2x - 440 &= 0
\end{align*}
\]

Find factors of \(-440\) whose sum is 2.

\(-20(22) = -440\) and \(-20 + 22 = 2\)

\[
x^2 - 20x + 22x - 440 = 0
\]

\[
x(x - 20) + 22(x - 20) = 0
\]

\[
(x + 22)(x - 20) = 0
\]

\[x = -22 \text{ or } x = 20\]

When \( x = -22 \), the dimensions of the rectangle becomes negative. So, \( x = 20 \).

The length of the rectangle is 24 ft and width is 18 ft.

**ANSWER:**
\( x = 20; \) 24 in. by 18 in.

**SOLUTION:**
The area of a rectangle of length \( l \) and width \( w \) is \( l \times w \).

Here, \( l = 3x - 4 \), \( w = x + 2 \), and area = 448.

\[
\begin{align*}
(3x - 4)(x + 2) &= 448 \\
3x^2 + 6x - 4x - 8 &= 448 \\
3x^2 + 2x - 456 &= 0
\end{align*}
\]

Find factors of \(3(-456) = -1368\) whose sum is 2.

\(-36(38) = -1368\) and \(-36 + 38 = 2\)

\[
3x^2 - 36x + 38x - 456 = 0
\]

\[
3x(x - 12) + 38(x - 12) = 0
\]

\[
(x - 12)(3x + 38) = 0
\]

\[x = 12 \text{ or } x = -\frac{38}{3}\]

When \( x = -\frac{38}{3} \), the dimensions of the rectangle become negative. So, \( x = 12 \).

The length of the rectangle is 32 ft and width is 14 ft.

**ANSWER:**
\( x = 12; \) 14 ft by 32 ft
4-3 Solving Quadratic Equations by Factoring

Solve each equation by factoring.

49. $12x^2 - 4x = 5$

SOLUTION:
Write the equation with right side equal to zero.
$12x^2 - 4x - 5 = 0$

Find factors of $12(-5) = -60$ whose sum is $-4$.
$-10(6) = -60$ and $-10 + 6 = -4$

$12x^2 - 10x + 6x - 5 = 0$
$2x(6x - 5) + 1(6x - 5) = 0$
$(6x - 5)(2x + 1) = 0$
$\Rightarrow 6x - 5 = 0$ or $2x + 1 = 0$
$\Rightarrow x = \frac{5}{6}$ or $x = -\frac{1}{2}$

Therefore, the roots are $\frac{5}{6}$ and $-\frac{1}{2}$.

ANSWER:
$-\frac{1}{2}, \frac{5}{6}$

50. $5x^2 = 15x$

SOLUTION:
Write the equation with right side equal to zero.
$5x^2 - 15x = 0$

Factor out the GCF of the left side, $5x$.
$5x(x - 3) = 0$

Use the Zero Product Property.
$5x(x - 3) = 0 \Rightarrow 5x = 0$ or $x - 3 = 0$
$\Rightarrow x = 0$ or $x = 3$

Therefore, the roots are $0$ and $3$.

ANSWER:
$0, 3$
51. \(16x^2 + 36 = -48x\)

**SOLUTION:**
Write the equation with right side equal to zero.

\(16x^2 + 48x + 36 = 0\)

Divide each side of the equation by 4.

\(4x^2 + 12x + 9 = 0\)

Find factors of 4(9) = 36 whose sum is 12.

\(6(6) = 36 \text{ and } 6 + 6 = 12\)

\[
4x^2 + 6x + 6x + 9 = 0
\]

\[
2x(2x + 3) + 3(2x + 3) = 0
\]

\[
(2x + 3)^2 = 0
\]

\[
\Rightarrow 2x + 3 = 0
\]

\[
\Rightarrow x = -\frac{3}{2}
\]

Therefore, the only repeated root is \(-\frac{3}{2}\).

**ANSWER:**

\[-\frac{3}{2}\]

52. \(75x^2 - 60x = -12\)

**SOLUTION:**
Write the equation with right side equal to zero.

\(75x^2 - 60x + 12 = 0\)

Divide each side of the equation by 3.

\(25x^2 - 20x + 4 = 0\)

Find factors of 25(4) = 100 whose sum is -20.

\(-10(-10) = 100 \text{ and } -10 + (-10) = -20\)

\[
25x^2 - 10x - 10x + 4 = 0
\]

\[
5x(5x - 2) - 2(5x - 2) = 0
\]

\[
(5x - 2)^2 = 0
\]

\[
\Rightarrow 5x - 2 = 0
\]

\[
\Rightarrow x = \frac{2}{5}
\]

Therefore, the only repeated root is \(\frac{2}{5}\).

**ANSWER:**

\[
\frac{2}{5}
\]
53. \(4x^2 - 144 = 0\)

**SOLUTION:**
Factor out the GCF of the left side, 4.

\[4\left(x^2 - 36\right) = 0\]

Use the identity \(a^2 - b^2 = (a + b)(a - b)\) to factor \(x^2 - 36\).

\[x^2 - 36 = (x + 6)(x - 6)\]

Use the Zero Product Property.

\[4(x + 6)(x - 6) = 0 \Rightarrow x + 6 = 0 \text{ or } x - 6 = 0\]

\[\Rightarrow x = -6 \text{ or } x = 6\]

Therefore, the roots are 6 and -6.

**ANSWER:**
6, -6

54. \(-7x + 6 = 20x^2\)

**SOLUTION:**
Write the equation with right side equal to zero.

\[20x^2 + 7x - 6 = 0\]

Find factors of \(20(-6) = -120\) whose sum is 7.

\[15(-8) = -120 \text{ and } 15 + (-8) = 7\]

\[20x^2 + 15x - 8x - 6 = 0\]

\[5x(4x + 3) - 2(4x + 3) = 0\]

\[(5x - 2)(4x + 3) = 0\]

\[\Rightarrow 5x - 2 = 0 \text{ or } 4x + 3 = 0\]

\[\Rightarrow x = \frac{2}{5} \text{ or } x = -\frac{3}{4}\]

Therefore, the roots are \(\frac{2}{5}\) and \(-\frac{3}{4}\).

**ANSWER:**
\(\frac{2}{5}, -\frac{3}{4}\)
55. **MOVIE THEATER** A company plans to build a large multiplex theater. The financial analyst told her manager that the profit function for their theater was \( P(x) = -x^2 + 48x - 512 \), where \( x \) is the number of movie screens, and \( P(x) \) is the profit earned in thousands of dollars. Determine the range of production of movie screens that will guarantee that the company will not lose money.

**SOLUTION:**
For the company not to lose money, the profit should be non-negative. That is, at least zero.

\[-x^2 + 48x - 512 = 0\]

Factor out \(-1\).

\[-1(x^2 - 48x + 512) = 0\]

Factor \( x^2 - 48x + 512 \).

Find factors of 512 whose sum is 48.

\[-16(-32) = 512 \text{ and } -16 + (-32) = -48\]

\[x^2 - 16x - 32x + 512 = 0\]

\[x(x - 16) - 32(x - 16) = 0\]

\[(x - 16)(x - 32) = 0\]

\[\Rightarrow x - 16 = 0 \text{ or } x - 32 = 0\]

\[\Rightarrow x = 16 \text{ or } x = 32\]

A total of 16 to 32 screens will guarantee that company will not lose money.

**ANSWER:**
16 to 32 screens

---

**Write a quadratic equation in standard form with the given root(s).**

56. \( \frac{-4}{7} \frac{3}{8} \)

**SOLUTION:**
Write the pattern.

\[(x - p)(x - q) = 0\]

Replace \( p \) and \( q \) with \( \frac{-4}{7} \) and \( \frac{3}{8} \).

\[\left( x - \left( \frac{-4}{7} \right) \right) \left( x - \frac{3}{8} \right) = \left( x + \frac{4}{7} \right) \left( x - \frac{3}{8} \right) = 0\]

Use the FOIL method to multiply.

\[x(x) + x \left( \frac{3}{8} \right) + \frac{4}{7} (x) + \frac{4}{7} \left( \frac{3}{8} \right) = 0\]

\[x^2 - \frac{3}{8} x + \frac{4}{7} x - \frac{12}{56} = 0\]

Multiply each side by 56.

\[56x^2 - 21x + 32x - 12 = 0\]

\[56x^2 + 11x - 12 = 0\]

**ANSWER:**

\[56x^2 + 11x - 12 = 0\]
4-3 Solving Quadratic Equations by Factoring

57. 3.4, 0.6

**SOLUTION:**
Write the pattern.

\[(x - p)(x - q) = 0\]

Replace \(p\) and \(q\) with 3.4 and 0.6.

\[(x - 3.4)(x - 0.6) = 0\]

Use the FOIL method to multiply.

\[
x(x) + x(-0.6) - 3.4(x) - 3.4(-0.6) = 0
\]

\[
x^2 - 0.6x - 3.4x + 2.04 = 0
\]

\[
x^2 - 4x + 2.04 = 0
\]

Multiply each side by 25.

\[25x^2 - 100x + 51 = 0\]

**ANSWER:**
\[25x^2 - 100x + 51 = 0\]

58. \(\frac{2}{11}\) and \(\frac{5}{9}\)

**SOLUTION:**
Write the pattern.

\[(x - p)(x - q) = 0\]

Replace \(p\) and \(q\) with \(\frac{2}{11}\) and \(\frac{5}{9}\).

\[
\left(x - \frac{2}{11}\right)\left(x - \frac{5}{9}\right) = 0
\]

Use the FOIL method to multiply.

\[
x(x) + x\left(-\frac{5}{9}\right) - \frac{2}{11}(x) - \frac{2}{11}\left(-\frac{5}{9}\right) = 0
\]

\[
x^2 - \frac{5}{9}x - \frac{2}{11}x + \frac{10}{99} = 0
\]

Multiply each side by 99.

\[99x^2 - 55x - 18x + 10 = 0\]

\[99x^2 - 73x + 10 = 0\]

**ANSWER:**
\[99x^2 - 73x + 10 = 0\]
4-3 Solving Quadratic Equations by Factoring

Solve each equation by factoring.

59. \(10x^2 + 25x = 15\)

\[10x^2 + 25x - 15 = 0\]

Divide each side by 5.

\[2x^2 + 5x - 3 = 0\]

Find factors of 2(−3) = −6 whose sum is 5.

\(-1(6) = -6\) and \(-1 + 6 = 5\)

\[2x^2 - x + 6x + 3 = 0\]

\[x(2x - 1) + 3(2x - 1) = 0\]

\[(2x - 1)(x + 3) = 0\]

\[2x - 1 = 0\] or \[x + 3 = 0\]

\[\Rightarrow x = \frac{1}{2}\] or \[x = -3\]

Therefore, the roots are \(-3\) and \(\frac{1}{2}\).

ANSWER:

\(-3, \frac{1}{2}\)

60. \(27x^2 + 5 = 48x\)

SOLUTION:
Write the equation with right side equal to zero.

\[27x^2 - 48x + 5 = 0\]

Find factors of 27(5) = 135 whose sum is −48.

\(-45(−3) = 135\) and \(-45 + (−3) = −48\)

\[\Rightarrow 27x^2 - 45x - 3x + 5 = 0\]

\[9x(3x - 5) - 1(3x - 5) = 0\]

\[(3x - 5)(9x - 1) = 0\]

\[\Rightarrow 3x - 5 = 0\] or \[9x - 1 = 0\]

\[\Rightarrow x = \frac{5}{3}\] or \[x = \frac{1}{9}\]

Therefore, the roots are \(\frac{5}{3}\) and \(\frac{1}{9}\).

ANSWER:

\[\frac{5}{3}, \frac{1}{9}\]
61. $x^2 + 0.25x = 1.25$

**SOLUTION:**
Write the equation with right side equal to zero.

$x^2 + 0.25x - 1.25 = 0$

Multiply each side by 4.

$4x^2 + x - 5 = 0$

Find factors of 4(–5) = –20 whose sum is 1.

5(–4) = –20 and 5 + (–4) = 1

$4x^2 - 4x + 5x - 5 = 0$

$4x(x - 1) + 5(x - 1) = 0$

$(4x + 1)(x - 1) = 0$

$\Rightarrow 4x + 1 = 0$ or $x - 1 = 0$

$\Rightarrow x = -\frac{1}{4}$ or $x = 1$

Therefore, the roots are 1 and $-\frac{5}{4}$.

**ANSWER:**
1, $-\frac{5}{4}$

62. $48x^2 - 15 = -22x$

**SOLUTION:**
Write the equation with right side equal to zero.

$48x^2 + 22x - 15 = 0$

Find factors of 48(–15) = –720 whose sum is 22.

$40(-18) = 720$ and $40 + (-18) = 22$

$48x^2 + 40x - 18x - 15 = 0$

$8x(6x + 5) - 3(6x + 5) = 0$

$(6x + 5)(8x - 3) = 0$

$\Rightarrow 6x + 5 = 0$ or $8x - 3 = 0$

$\Rightarrow x = -\frac{5}{6}$ or $x = \frac{3}{8}$

Therefore, the roots are $-\frac{5}{6}$ and $\frac{3}{8}$.

**ANSWER:**
$\frac{3}{8}, \frac{5}{6}$
4-3 Solving Quadratic Equations by Factoring

63. \(3x^2 + 2x = 3.75\)

**SOLUTION:**
Write the equation with right side equal to zero.

\[3x^2 + 2x - 3.75 = 0\]

Multiply each side by 4.

\[12x^2 + 8x - 15 = 0\]

Find factors of \(12(-15) = -180\) whose sum is 8.

\[18(-10) = 8 \text{ and } 18 + (-10) = 8\]

\[12x^2 + 18x - 10x - 15 = 0\]

\[6x(2x + 3) - 5(2x + 3) = 0\]

\[(6x - 5)(2x + 3) = 0\]

\[6x - 5 = 0 \text{ or } 2x + 3 = 0\]

\[x = \frac{5}{6} \text{ or } x = -\frac{3}{2}\]

Therefore, the roots are \(\frac{5}{6}\) and \(-\frac{3}{2}\).

**ANSWER:**

\[
\begin{array}{c}
\frac{3}{6} \\
\frac{5}{6}
\end{array}
\]

64. \(-32x^2 + 56x = 12\)

**SOLUTION:**
Write the equation with right side equal to zero.

\[32x^2 - 56x + 12 = 0\]

Divide each side by 4.

\[8x^2 - 14x + 3 = 0\]

Find factors of \(8(3) = 24\) whose sum is \(-14\).

\[-12(-2) = 24 \text{ and } -12 + (-2) = -14\]

\[8x^2 - 12x - 2x + 3 = 0\]

\[4x(2x - 3) - 1(2x - 3) = 0\]

\[(4x - 1)(2x - 3) = 0\]

\[4x - 1 = 0 \text{ or } 2x - 3 = 0\]

\[x = \frac{1}{4} \text{ or } x = \frac{3}{2}\]

Therefore, the roots are \(\frac{1}{4}\) and \(\frac{3}{2}\).

**ANSWER:**

\[
\begin{array}{c}
\frac{1}{4} \\
\frac{3}{2}
\end{array}
\]
4-3 Solving Quadratic Equations by Factoring

65. DESIGN A square is cut out of the figure at the right. Write an expression for the area of the figure that remains, and then factor the expression.

![Diagram of a square with a smaller square cut out]

SOLUTION:
The area of the figure that remains is the difference between the areas of the square with side \(x\) units and that of the square with side 6 units.

\[
x^2 - (6)^2
\]

Use the identity \(a^2 - b^2 = (a + b)(a - b)\) to factor \(x^2 - 36\).

\[
x^2 - (6)^2 = (x + 6)(x - 6)
\]

ANSWER:
\(x^2 - 6^2; (x + 6)(x - 6)\)

66. CCSS PERSEVERANCE After analyzing the market, a company that sells Web sites determined the profitability of their product was modeled by \(P(x) = -16x^2 + 368x - 2035\), where \(x\) is the price of each Web site and \(P(x)\) is the company’s profit. Determine the price range of the Web sites that will be profitable for the company.

SOLUTION:
For the company not to lose money, the profit should be non-negative. That is, at least zero.

\[-16x^2 + 368x - 2035 = 0\]

Factor out -1.

\[-1(16x^2 - 368x + 2035) = 0\]

\[16x^2 - 368x + 2035 = 0\]

Factor 16\(x^2 - 368x + 2035\).

Find factors of 16(2035) = 32560 whose sum is -368.

\[-220(-148) = 32560\text{ and }-220 + (-148) = -368\]

\[16x^2 - 220x - 148x + 2035 = 0\]

\[4x(4x - 55) - 37(4x - 55) = 0\]

\[(4x - 55)(4x - 37) = 0\]

\[4x - 55 = 0\text{ or }4x - 37 = 0\]

\[\Rightarrow x = \frac{55}{4} = 13\frac{3}{4}\text{ or }x = \frac{37}{4} = 9\frac{1}{4}\]

A price range of $9.25 to $13.75 will be profitable for the company.

ANSWER:
$9.25 to $13.75

67. PAINTINGS Enola wants to add a border to her painting, distributed evenly, that has the same area as the painting itself. What are the dimensions of the painting with the border included?
**4-3 Solving Quadratic Equations by Factoring**

**SOLUTION:**
The area of a rectangle of length \( l \) and width \( w \) is \( l \times w \).

Here, \( l = 15 \text{ in} \), \( w = 10 \text{ in} \). So, area = 150 in\(^2\).

The area including the border will be double the area of the painting. So, the total area will be 300 in\(^2\).

Let \( x \) be the amount in length and width of the border. Then,
\[
(x + 15)(x + 10) = 300
\]

Use the FOIL method to multiply the left.
\[
x(x) + x(10) + 15(x) + 15(10) = 300
\]
\[
x^2 + 10x + 15x + 150 = 300
\]
\[
x^2 + 25x - 150 = 0
\]

Find the factors of -150 whose sum is 25.

\(-5(30) = -150 \text{ and } -5 + 30 = 25\)

Write 25\(x \) as \(-5x + 30\(x \).

\[
x^2 - 5x + 30x - 150 = 0
\]
\[
x(x - 5) + 30(x - 5) = 0
\]
\[
(x - 5)(x + 30) = 0
\]

Use the Zero Product Property.

\[
(x - 5)(x + 30) = 0 \Rightarrow x - 5 = 0 \text{ or } x + 30 = 0
\]
\[
\Rightarrow x = 5 \text{ or } x = -30
\]

So, the roots are 5 and -30.

But \( x \) is a length, so it cannot be negative. So, \( x = 5 \).

Therefore, the dimensions of the painting, including the border are 20 in by 15 in.

**ANSWER:**
20 in. by 15 in.

68. **MULTIPLE REPRESENTATIONS** In this problem, you will consider \( a(x - p)(x - q) = 0 \).

a. **GRAPHICAL** Graph the related function for \( a = 1, p = 2, \) and \( q = -3 \).

b. **ANALYTICAL** What are the solutions of the equation?

c. **GRAPHICAL** Graph the related functions for \( a = 4, -3, \) and \( \frac{1}{2} \) on the same graph.

d. **VERBAL** What are the similarities and differences between the graphs?

e. **VERBAL** What conclusion can you make about the relationship between the factored form of a quadratic equation and its solutions?

**SOLUTION:**
a. When \( a = 1, p = 2, \) and \( q = -3 \), the function becomes,
\[
(x - 2)(x + 3) = 0
\]
That is, \( x^2 + x - 6 = 0 \).
Graph the function on a coordinate plane.
b. The graph intersects the x-axis at -3 and 2. So, the solutions are \( x = -3 \) and \( x = 2 \).

c. When \( a = 4 \), the function is \( 4x^2 + 4x - 24 = 0 \).

When \( a = -3 \), it is \( -3x^2 - 3x + 18 = 0 \) and when \( a = \frac{1}{2} \), it is \( \frac{1}{2}x^2 + \frac{1}{2}x - 3 = 0 \).

Draw the four graphs on the same coordinate plane.

d. Sample answer: They all have the same roots, \( p \) and \( q \) which are 2 and -3 respectively. Therefore, they all have the same solutions. The graphs are shaped differently due to the value of \( a \). The graph with \( a = -3 \) is flipped due to the negative.

e. When quadratic equations have the same factors, they will have the same solutions, regardless of the value of \( a \), which only affects the shape of the graphs.

ANSWER:

a.

b. 2 and -3

c.

69. GEOMETRY The area of the triangle is 26 square centimeters. Find the length of the base.

SOLUTION:
The area of a triangle of base \( b \) and height \( h \) is given by the formula \( \frac{1}{2}bh \).

Here, \( b = x + 7 \), \( h = x - 2 \), and area = 26 cm\(^2\).

\[
\frac{1}{2}(x+7)(x-2) = 26
\]

\[
(x+7)(x-2) = 52
\]

\[
x^2 + 5x - 14 = 52
\]

\[
x^2 + 5x - 66 = 0
\]

Find the factors of -66 whose sum is +5.

\((-6)(11) = -66\) and \((-6) + (11) = 5\)

Write \( 5x \) as \((-6x) + (11x)\).
4-3 Solving Quadratic Equations by Factoring

\[ x^2 - 6x + 11x - 66 = 0 \]
\[ x(x - 6) + 11(x - 6) = 0 \]
\[ (x + 11)(x - 6) = 0 \]

Use the Zero Product Property.

\[ (x + 11)(x - 6) = 0 \Rightarrow x + 11 = 0 \quad \text{or} \quad x - 6 = 0 \]
\[ x = -11 \quad \text{or} \quad x = 6 \]

So, the roots are -11 and 6.

But when \( x = -11 \), the height of the triangle becomes negative. So, \( x = 6 \).

Therefore, the length of the base of the triangle is 13 cm.

**ANSWER:**
13 cm

70. **SOCCER** When a ball is kicked in the air, its height in meters above the ground can be modeled by \( h(t) = -4.9t^2 + 14.7t \) and the distance it travels can be modeled by \( d(t) = 16t \), where \( t \) is the time in seconds.

a. How long was the ball in the air?

b. How far did it travel before it hit the ground? (Hint: Ignore air resistance.)

c. What was the maximum height of the ball?

**SOLUTION:**

a. When the ball hits the ground, the height will be zero.

That is, \( h(t) = 0 \).

\[-4.9t^2 + 14.7t = 0 \]

Solve for \( t \) to find the time that the ball was in air.

\[ t(-4.9t + 14.7) = 0 \]
\[ \Rightarrow t = 0 \quad \text{or} \quad -4.9t + 14.7 = 0 \]
\[ \Rightarrow t = 0 \quad \text{or} \quad t = \frac{-14.7}{-4.9} = 3 \]

The solution \( t = 0 \) is not valid as the ball has already been hit. Therefore, the ball was in air for 3 seconds.

b. Substitute \( t = 3 \) in the formula to find the distance traveled by the ball.

\[ d(3) = 16(3) \]
\[ = 48 \]

Therefore, the ball will travel 48 m before it hits the ground.

c. The maximum height is the \( y \)-coordinate of the vertex of the parabola formed by the equation \( h(t) = -4.9t^2 + 14.7t \). The \( y \)-coordinate of the vertex of a parabola is given by \( -\frac{b^2 - 4ac}{4a} \).

Here, \( a = -4.9, b = 14.7 \) and \( c = 0 \). So, the \( y \)-coordinate of the vertex is

\[ \frac{(14.7)^2 - 4(-4.9)(0)}{4(-4.9)} = \frac{216.09}{19.6} = 11.025. \]

Therefore, the maximum height of the ball is 11.025 m.

**ANSWER:**

a. 3 seconds
b. 48 m
c. 11.025 m
4-3 Solving Quadratic Equations by Factoring

Factor each polynomial.

71. $18a - 24ay + 48b - 64by$

**SOLUTION:**
Factor $3a$ from the first two terms and $8b$ from the last two terms.

$$18a - 24ay + 48b - 64by = 3a(6 - 8y) + 8b(6 - 8y)$$

Factor $6 - 8y$ from the two terms.

$$3a(6 - 8y) + 8b(6 - 8y) = (6 - 8y)(3a + 8b) = 2(3 - 4y)(3a + 8b)$$

**ANSWER:**
$2(3 - 4y)(3a + 8b)$

72. $3x^2 + 2xy + 10y + 15x$

**SOLUTION:**
Factor $x$ from the first two terms and $5$ from the last two terms.

$$3x^2 + 2xy + 10y + 15x = x(3x + 2y) + 5(2y + 3x)$$

$$= x(3x + 2y) + 5(3x + 2y)$$

Factor $3x + 2y$ from the two terms.

$$x(3x + 2y) + 5(3x + 2y) = (3x + 2y)(x + 5)$$

**ANSWER:**
$(3x + 2y)(x + 5)$

73. $6a^2b^2 - 12ab^2 - 18b^3$

**SOLUTION:**
Factor $6b^2$ from all the three terms.

$$6a^2b^2 - 12ab^2 - 18b^3 = 6b^2(a^2) - 6b^2(2a) - 6b^2(3b)$$

$$= 6b^2(a^2 - 2a - 3b)$$

**ANSWER:**
$6b^2(a^2 - 2a - 3b)$

74. $12a^2 - 18ab + 30ab^3$

**SOLUTION:**
Factor $6a$ from all the three terms.

$$12a^2 - 18ab + 30ab^3 = 6a(2a) - 6a(3b) - 6a(5b^3)$$

$$= 6b^2(2a - 3b - 5b^3)$$

**ANSWER:**
$6a(2a - 3b + 5b^3)$
4-3 Solving Quadratic Equations by Factoring

75. $32ax + 12bx - 48ay - 18by$

**SOLUTION:**
Factor $4x$ from the first two terms and $-6y$ from the last two terms.

$32ax + 12bx - 48ay - 18by = 4x(8a + 3b) - 6y(8a + 3b)$

Factor $8a + 3b$ from the two terms.

$4x(8a + 3b) - 6y(8a + 3b) = (8a + 3b)(4x - 6y)$

$= (8a + 3b)(2)(2x - 3y)$

$= 2(8a + 3b)(2x - 3y)$

**ANSWER:**
$2(2x - 3y)(8a + 3b)$

76. $30ac + 80bd + 40ad + 60bc$

**SOLUTION:**
Rearrange the terms to group the terms with common factors.

$30ac + 80bd + 40ad + 60bc = 30(ac + 40ad + 60bc + 80bd)$

Factor $10a$ from the first two terms and $20b$ from the last two terms.

$30ac + 40ad + 60bc + 80bd = 10a(3c + 4d) + 20b(3c + 4d)$

Factor $3c + 4d$ from the two terms.

$10a(3c + 4d) + 20b(3c + 4d) = (3c + 4d)(10a + 20b)$

$= (3c + 4d)(10)(a + 2b)$

$= 10(a + 2b)(3c + 4d)$

**ANSWER:**
$10(a + 2b)(3c + 4d)$
4-3 Solving Quadratic Equations by Factoring

77. \(5ax^2 - 2by^2 - 5ay^2 + 2bx^2\)

**SOLUTION:**
Rearrange the terms to group the terms with common factors.

\[
5ax^2 - 2by^2 - 5ay^2 + 2bx^2 = 5ax^2 - 5ay^2 + 2bx^2 - 2by^2
\]

Factor 5a from the first two terms and 2b from the last two terms.

\[
5ax^2 - 5ay^2 + 2bx^2 - 2by^2 = 5a(x^2 - y^2) + 2b(x^2 - y^2)
\]

Factor \(x^2 - y^2\) from the two terms.

\[
5a(x^2 - y^2) + 2b(x^2 - y^2) = (x^2 - y^2)(5a + 2b)
\]

\[
= (x + y)(x - y)(5a + 2b)
\]

**ANSWER:**
\((x + y)(x - y)(5a + 2b)\)

78. \(12c^2x + 4d^2y - 3d^2x - 16c^2y\)

**SOLUTION:**
Rearrange the terms to group the terms with common factors.

\[
12c^2x + 4d^2y - 3d^2x - 16c^2y = 12c^2x - 16c^2y - 3d^2x + 4d^2y
\]

Factor 4c² from the first two terms and -d² from the last two terms.

\[
12c^2x - 16c^2y - 3d^2x + 4d^2y = 4c^2(3x - 4y) - d^2(3x - 4y)
\]

Factor 3x - 4y from the two terms.

\[
4c^2(3x - 4y) - d^2(3x - 4y) = (3x - 4y)(4c^2 - d^2)
\]

\[
= (3x - 4y)(2c + d)(2c - d)
\]

**ANSWER:**
\((2c + d)(2c - d)(3x - 4y)\)
4-3 Solving Quadratic Equations by Factoring

79. **ERROR ANALYSIS** Gwen and Morgan are solving \(-12x^2 + 5x + 2 = 0\). Is either of them correct? Explain your reasoning.

\[
\begin{align*}
\text{Gwen} & \quad -12x^2 + 5x + 2 = 0 \\
& \quad -12x^2 + 8x - 3x + 2 = 0 \\
& \quad 4x(-3x + 2) - (3x + 2) = 0 \\
& \quad (4x - 1)(3x + 2) = 0 \\
& \quad x = \frac{1}{4} \text{ or } -\frac{2}{3} \\
\end{align*}
\]

\[
\begin{align*}
\text{Morgan} & \quad -12x^2 + 5x + 2 = 0 \\
& \quad -12x^2 + 8x - 3x + 2 = 0 \\
& \quad 4x(-3x + 2) + (3x + 2) = 0 \\
& \quad (4x + 1)(3x + 2) = 0 \\
& \quad x = -\frac{1}{4} \text{ or } \frac{2}{3} \\
\end{align*}
\]

**SOLUTION:**
Morgan is correct. In step 3, Gwen did not have like terms in the parentheses in the third line.

**ANSWER:**
Sample answer: Morgan; Gwen did not have like terms in the parentheses in the third line.

80. **CHALLENGE** Solve \(3x^6 - 39x^4 + 108x^2 = 0\) by factoring.

**SOLUTION:**
Substitute \(x^2 = X\). Then the equation becomes

\[3X^3 - 39X^2 + 108X = 0.\]

Factor out \(X\) from the three terms.

\[X \left(3X^2 - 39X + 108\right) = 0\]

By the Zero Product Property, either \(X = 0\) or \(3X^2 - 39X + 108 = 0\).

\[X = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0\]

Solve the equation \(3X^2 - 39X + 108 = 0\).

Find factors of \(3(108) = 324\) whose sum is \(-39\).

\(-12(-27) = 324\) and \(-12 + (-27) = -39\)

\[3X^2 - 12X - 27X + 108 = 0\]

\[3X(X - 4) - 27(X - 4) = 0\]

\[(3X - 27)(X - 4) = 0\]

\(\Rightarrow 3X - 27 = 0 \text{ or } X - 4 = 0\)

\(\Rightarrow X = 9 \text{ or } X = 4\)

\[X = 9 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3\]

\[X = 4 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2\]

Therefore, the roots are 0, 3, \(-3\), 2, or \(-2\).

**ANSWER:**
0, 3, \(-3\), 2, or \(-2\)
4-3 Solving Quadratic Equations by Factoring

81. **CHALLENGE** The rule for factoring a difference of cubes is shown below. Use this rule to factor $40x^5 - 135x^2y^3$.

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

**SOLUTION:**
First factor out the GCF $5x^2$ from the two terms.

$$40x^5 - 135x^2y^3 = 5x^2(8x^3 - 27y^3)$$

$$40x^5 - 135x^2y^3 = 5x^2\left((2x)^3 - (3y)^3\right)$$

Here, $a = 2x$ and $b = 3y$. Use the rule to factor $(2x)^3 - (3y)^3$.

$$(2x)^3 - (3y)^3 = (2x - 3y)(4x^2 + 6xy + 9y^2)$$

Therefore,

$$40x^5 - 135x^2y^3 = 5x^2(2x - 3y)(4x^2 + 6xy + 9y^2).$$

**ANSWER:**

$$5x^2(2x - 3y)(4x^2 + 6xy + 9y^2)$$

82. **OPEN ENDED** Choose two integers. Then write an equation in standard form with those roots. How would the equation change if the signs of the two roots were switched?

**SOLUTION:**
Sample answer: $3$ and $6 \rightarrow x^2 - 9x + 18 = 0. -3$ and $-6 \rightarrow x^2 + 9x + 18 = 0$. The linear term changes sign.

**ANSWER:**
Sample answer: $3$ and $6 \rightarrow x^2 - 9x + 18 = 0. -3$ and $-6 \rightarrow x^2 + 9x + 18 = 0$. The linear term changes sign.

83. **CHALLENGE** For a quadratic equation of the form $(x - p)(x - q) = 0$, show that the axis of symmetry of the related quadratic function is located halfway between the $x$-intercepts $p$ and $q$.

**SOLUTION:**
Sample answer:
Original equation is $(x - p)(x - q) = 0$.

Multiply.

$$x^2 - px - qx + pq = 0$$

Simplify.

$$x^2 - (p + q)x + pq = 0$$

The formula for axis of symmetry is $x = \frac{-b}{2a}$.

We have $a = 1$ and $b = -(p + q)$.

$$x = \frac{-(p + q)}{2(1)}$$

Simplify.

$$x = \frac{p + q}{2}$$
4-3 Solving Quadratic Equations by Factoring

By the definition of midpoint, \( x \) is midway between \( p \) and \( q \).

**ANSWER:**
Sample answer:

\[(x - p)(x - q) = 0 \]

**SOLUTION:**
Write the pattern.

\[x = \frac{-b}{2a} \]

**Formula for axis of symmetry**

\[x = \frac{-(p + q)}{2(1)} \]

\[a = 1 \text{ and } b = - \]

\[(p + q) \]

**Simplify.**

\[x = \frac{p + q}{2} \]

**Simplify.**

\( x \) is midway between \( p \) and \( q \). **Definition of midpoint**

84. **WRITE A QUESTION** A classmate is using the guess and check strategy to factor trinomials of the form \( x^2 + bx + c \). Write a question to help him think of a way to use that strategy for \( ax^2 + bx + c \).

**SOLUTION:**
Sample answer: What do you know about \( a \cdot c \) to use guess and check to factor trinomials of the form \( ax^2 + bx + c \)?

**ANSWER:**
Sample answer: What do you know about \( a \cdot c \) to use guess and check to factor trinomials of the form \( ax^2 + bx + c \)?

85. **CCSS ARGUMENTS** Determine whether the following statement is sometimes, always, or never true. Explain your reasoning.

In a quadratic equation in standard form where \( a, b, \) and \( c \) are integers, if \( b \) is odd, then the quadratic cannot be a perfect square trinomial.

**SOLUTION:**
Sample answer: Always; in order to factor using perfect square trinomials, the coefficient of the linear term, \( bx \), must be a multiple of 2, or even.

**ANSWER:**
Sample answer: Always; in order to factor using perfect square trinomials, the coefficient of the linear term, \( bx \), must be a multiple of 2, or even.
4-3 Solving Quadratic Equations by Factoring

86. **WRITING IN MATH** Explain how to factor a trinomial in standard form with \( a > 1 \).

**SOLUTION:**
Sample answer: In standard form, we have \( ax^2 + bx + c \). Multiply \( a \) and \( c \).

Then find a pair of integers, \( g \) and \( h \), that multiply to equal \( ac \) and add to equal \( b \).

Then write out the quadratic, turning the middle term, \( bx \), into \( gx + hx \).

We now have \( ax^2 + gx + hx + c \).

Now factor the GCF from the first two terms and then factor the GCF from the second two terms.

So we now have \( \text{GCF}(x - q) + \text{GCF}_2(x - q) \).

Simplifying, we get \( (\text{GCF} + \text{GCF}_2)(x - q) \) or \((x - p)(x - q)\).

**ANSWER:**
Sample answer: In standard form, we have \( ax^2 + bx + c \). Multiply \( a \) and \( c \).

Then find a pair of integers, \( g \) and \( h \), that multiply to equal \( ac \) and add to equal \( b \).

Then write out the quadratic, turning the middle term, \( bx \), into \( gx + hx \).

We now have \( ax^2 + gx + hx + c \).

Now factor the GCF from the first two terms and then factor the GCF from the second two terms.

So we now have \( \text{GCF}(x - q) + \text{GCF}_2(x - q) \).

Simplifying, we get \( (\text{GCF} + \text{GCF}_2)(x - q) \) or \((x - p)(x - q)\).

87. **SHORT RESPONSE** If \( ABCD \) is transformed by \((x, y) \to (3x, 4y)\), determine the area of \( A'B'C'D' \).

**SOLUTION:**
Here, \( A'B'C'D' \) will be a rectangle.

Determine the length and width of \( A'B'C'D' \).

The coordinates of \( A' \) will remain \((0, 0)\) and the coordinates of \( B' \) will be \( (3(4), 4(0)) = (12, 0) \).

So, the width of the transformed rectangle will be 12.

Now, the coordinates of \( C' \) will be \( (3(4), 4(4)) = (12, 16) \).

So, the length of the transformed rectangle will be 16.

Therefore, the area of \( A'B'C'D' \) is \( 12 \times 16 = 192 \) square units.

**ANSWER:**
192 square units

88. For \( y = 2|6 - 3x| + 4 \), which set describes \( x \) when \( y < 6 \)?

\[ A \left\{ \begin{array}{l} x < \frac{5}{3} \\ \frac{5}{3} < x < \frac{7}{3} \end{array} \right\} \]

\[ B \left\{ \begin{array}{l} x < \frac{5}{3} \\ \frac{7}{3} < x \end{array} \right\} \]
4-3 Solving Quadratic Equations by Factoring

C \(\left\{ x \mid x \leq \frac{5}{3}\right\}\)

D \(\left\{ x \mid x > \frac{7}{3}\right\}\)

**SOLUTION:**
When \(y < 6\), \(2|6-3x| + 4 < 6\).

Subtract 4 from each side of the inequality and divide by 2.
\(2|6-3x| < 2\)
\(|6-3x| < 1\)

When \(|a| < b\), then \(-b < a < b\). So, \(-1 < 6 - 3x < 1\).

Subtract 6 from each side.
\(-7 < -3x < -5\)

Divide each part by \(-3\). When you divide each side of an inequality by a negative number, the inequality sign should be reversed.
\(\frac{7}{3} > x > \frac{5}{3}\) or \(\frac{5}{3} < x < \frac{7}{3}\)

So, the solution set is \(\left\{ x \mid \frac{5}{3} < x < \frac{7}{3}\right\}\).

Therefore, the correct choice is A.

**ANSWER:**
A

89. **PROBABILITY** A 5-character password can contain the numbers 0 through 9 and 26 letters of the alphabet. None of the characters can be repeated. What is the probability that the password begins with a consonant?

F \(\frac{21}{26}\)

G \(\frac{21}{35}\)

H \(\frac{21}{36}\)

J \(\frac{5}{36}\)

**SOLUTION:**
There are \(26 + 10 = 36\) possible choices for the first character, and there are 21 consonants. So, the total number of outcomes is 36 and the number of favorable outcomes is 21.

\(P = \frac{21}{36}\)

Therefore, the correct choice is H.

**ANSWER:**
H
4-3 Solving Quadratic Equations by Factoring

90. SAT/ACT If \( c = \frac{8a^3}{b} \), what happens to the value of \( c \) when both \( a \) and \( b \) are doubled?

A) \( c \) is unchanged.
B) \( c \) is halved.
C) \( c \) is doubled.
D) \( c \) is multiplied by 4.
E) \( c \) is multiplied by 8.

**SOLUTION:**
When \( a \) is doubled, the value of \( a^3 \) becomes 8 times the original value of \( a \). So, when the values of both \( a \) and \( b \) are doubled, the value of \( c \) gets multiplied by \( \frac{8}{2} = 4 \). Therefore, the correct choice is D.

**ANSWER:**
D

**Use the related graph of each equation to determine its solutions.**

91. \( x^2 - 2x - 8 = 0 \)

**SOLUTION:**
The graph intersects the \( x \)-axis at –2 and 4. Therefore, the roots of the equation are –2 and 4.

**ANSWER:**
\(-2, 4\)

92. \( x^2 + 4x = 12 \)

**SOLUTION:**
The graph intersects the \( x \)-axis at –6 and 2. Therefore, the roots of the equation are –6 and 2.

**ANSWER:**
\(-6, 2\)
93. \( x^2 + 4x + 4 = 0 \)

**SOLUTION:**
The graph intersects the \( x \)-axis at \(-2\). Therefore, the root of the equation is \(-2\) and it is a repeated root.

**ANSWER:**
\(-2\)

Graph each function.

94. \( f(x) = x^2 - 6x + 2 \)

**SOLUTION:**
Make a table of values and then graph the function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>-6</td>
</tr>
<tr>
<td>3</td>
<td>-7</td>
</tr>
<tr>
<td>4</td>
<td>-6</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

**ANSWER:**
4-3 Solving Quadratic Equations by Factoring

95. \( f(x) = -2x^2 + 4x + 1 \)

**SOLUTION:**
Make a table of values and then graph the function.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-5</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-5</td>
</tr>
</tbody>
</table>

**ANSWER:**

96. \( f(x) = (x - 3)(x + 4) \)

**SOLUTION:**
Multiply the expression to get \( f(x) = x^2 + x - 12 \). Then make a table of values and then graph the function.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>-10</td>
</tr>
<tr>
<td>0</td>
<td>-12</td>
</tr>
<tr>
<td>2</td>
<td>-6</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

**ANSWER:**

97. **FUNDRAISING** Lawrence High School sold wrapping paper and boxed cards for their fundraising event. The school gets $1.00 for each roll of wrapping paper sold and $0.50 for each box of cards sold.
4-3 Solving Quadratic Equations by Factoring

<table>
<thead>
<tr>
<th>Total Amounts for Each Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class</td>
</tr>
<tr>
<td>Freshmen</td>
</tr>
<tr>
<td>Sophomores</td>
</tr>
<tr>
<td>Juniors</td>
</tr>
<tr>
<td>Seniors</td>
</tr>
</tbody>
</table>

a. Write a matrix that represents the amounts sold for each class and a matrix that represents the amount of money the school earns for each item sold.

b. Write a matrix that shows how much each class earned.

c. Which class earned the most money?

d. What is the total amount of money the school made from the fundraiser?

**SOLUTION:**

a. Let the first column represent the number of wrapping papers sold by each class and the second column represent the number of cards. Then the matrix can be written as:

\[
\begin{bmatrix}
72 & 49 \\
68 & 63 \\
90 & 56 \\
86 & 62
\end{bmatrix}
\]

The school gets $1.00 for each roll of wrapping paper sold and $0.50 for each box of cards sold. Then the matrix can be written as:

\[
\begin{bmatrix}
1.00 \\
0.50
\end{bmatrix}
\]

b. Multiply the matrices to find the matrix that shows how much each class earned.

c. The “juniors” class earned the most money.

d. Add the money earned by each class to find the total money earned by the school. $96.50 + 99.50 + 118.00 + 117.00 = 431.

The school made the total amount of $431 from the fundraiser.

**ANSWER:**

\[
\begin{bmatrix}
72 & 49 \\
68 & 63 \\
90 & 56 \\
86 & 62
\end{bmatrix}
\]

\[
\begin{bmatrix}
1.00 \\
0.50
\end{bmatrix}
\]

\[
\begin{bmatrix}
96.50 \\
99.50 \\
118.00 \\
117.00
\end{bmatrix}
\]

c. juniors

d. $431
4-3 Solving Quadratic Equations by Factoring

Simplify.

98. \( \sqrt{5} \cdot \sqrt{15} \)

\[ \text{SOLUTION:} \]
\[ \sqrt{5} \cdot \sqrt{15} = \sqrt{5 \cdot 5 \cdot 3} \]
\[ = \sqrt{5} \cdot \sqrt{5} \cdot \sqrt{3} \]
\[ = 5 \sqrt{3} \]

\[ \text{ANSWER:} \]
\[ 5 \sqrt{3} \]

99. \( \sqrt{8} \cdot \sqrt{32} \)

\[ \text{SOLUTION:} \]
\[ \sqrt{8} \cdot \sqrt{32} = \sqrt{(2)(2)(2)(2)(2)(2)} \]
\[ = \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2} \]
\[ = (\sqrt{2} \cdot \sqrt{2})(\sqrt{2} \cdot \sqrt{2})(\sqrt{2} \cdot \sqrt{2}) \]
\[ = 2 \cdot 2 \cdot 2 \]
\[ = 16 \]

\[ \text{ANSWER:} \]
\[ 16 \]

100. \( 2\sqrt{3} \cdot 2\sqrt{9} \)

\[ \text{SOLUTION:} \]
\[ 2\sqrt{3} \cdot 2\sqrt{9} = 2\sqrt{3} \cdot \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{3} \]
\[ = 2\sqrt{3} \cdot \sqrt{3} \cdot \sqrt{3} \]
\[ = 2(\sqrt{3} \cdot \sqrt{3})(\sqrt{3} \cdot \sqrt{3}) \]
\[ = 2 \cdot 3 \cdot 3 \]
\[ = 18 \]

\[ \text{ANSWER:} \]
\[ 18 \]