4-4 Complex Numbers

Simplify.

1. \(\sqrt{-81}\)

\[\text{SOLUTION:} \quad \sqrt{-81} = \sqrt{-1} \cdot 9 \]
\[= \sqrt{-1} \cdot 9i \]
\[= 9i \]

\[\text{ANSWER:} \quad 9i \]

2. \(\sqrt{-32}\)

\[\text{SOLUTION:} \quad \sqrt{-32} = \sqrt{-1} \cdot 2 \cdot 2 \cdot 2 \cdot 2 \]
\[= \sqrt{-1} \cdot 2^2 \cdot 2 \cdot 2 \]
\[= 4i \sqrt{2} \]

\[\text{ANSWER:} \quad 4i \sqrt{2} \]

3. \((4i)(-3i)\)

\[\text{SOLUTION:} \quad (4i)(-3i) = -12i^2 \]
\[= -12(-1) \]
\[= 12 \]

\[\text{ANSWER:} \quad 12 \]

4. \(3\sqrt{-24} \cdot 2\sqrt{-18}\)

\[\text{SOLUTION:} \quad 3\sqrt{-24} \cdot 2\sqrt{-18} \]
\[= 3 \cdot \sqrt{-1} \cdot 2 \cdot 2 \cdot 3 \cdot 2 \cdot \sqrt{-1} \cdot 2 \cdot 3 \]
\[= 3 \cdot i \cdot 2 \cdot \sqrt{2} \cdot \sqrt{3} \cdot 2 \cdot i \cdot \sqrt{2} \cdot 3 \]
\[= 72 \cdot i^2 \cdot \sqrt{3} \]
\[= 72 \cdot (-1) \cdot \sqrt{3} \]
\[= -72\sqrt{3} \]

\[\text{ANSWER:} \quad -72\sqrt{3} \]

5. \(i^{40}\)

\[\text{SOLUTION:} \quad i^{40} = (i^2)^{20} \]
\[= (-1)^{20} \]
\[= 1 \]

\[\text{ANSWER:} \quad 1 \]

6. \(i^{63}\)

\[\text{SOLUTION:} \quad i^{63} = i^{62} \cdot i \]
\[= (i^2)^{31} \cdot i \]
\[= -1 \cdot i \]
\[= -i \]

\[\text{ANSWER:} \quad -i \]
Simplify.

1. SOLUTION:

\[ 4x^2 + 32 = 0 \]

\[ 4x^2 = -32 \]
\[ x^2 = -8 \]
\[ x = \pm \sqrt{-8} \]
\[ x = \pm \sqrt{-1 \cdot 2 \cdot 2 \cdot 2} \]
\[ x = \pm 2i \sqrt{2} \]

ANSWER:
\[ \pm 2i \sqrt{2} \]

2. SOLUTION:

\[ x^2 + 1 = 0 \]

\[ x^2 = -1 \]
\[ x = \pm \sqrt{-1} \]
\[ x = \pm i \sqrt{1} \]
\[ x = \pm i \]

ANSWER:
\[ \pm i \]

Find the values of \( a \) and \( b \) that make each equation true.

9. \[ 3a + (4b + 2)i = 9 - 6i \]

SOLUTION:
Set the real parts equal to each other.
\[ 3a = 9 \]
\[ a = 3 \]
Set the imaginary parts equal to each other.
\[ 4b + 2 = -6 \]
\[ 4b = -8 \]
\[ b = -2 \]

ANSWER:
\[ 3, -2 \]

10. \[ 4b - 5 + (-a - 3)i = 7 - 8i \]

SOLUTION:
Set the real parts equal to each other.
\[ 4b - 5 = 7 \]
\[ 4b = 12 \]
\[ b = 3 \]
Set the imaginary parts equal to each other.
\[ -a - 3 = -8 \]
\[ -a = -5 \]
\[ a = 5 \]

ANSWER:
\[ 5, 3 \]
4-4 Complex Numbers

Simplify.

11. \((-1 + 5i) + (-2 - 3i)\)

**SOLUTION:**
\((-1 + 5i) + (-2 - 3i) = (-1 - 2) + (5i - 3i)
= -3 + 2i\)

**ANSWER:**
\(-3 + 2i\)

12. \((7 + 4i) - (1 + 2i)\)

**SOLUTION:**
\((7 + 4i) - (1 + 2i) = 7 + 4i - 1 - 2i
= 6 + 2i\)

**ANSWER:**
\(6 + 2i\)

13. \((6 - 8i)(9 + 2i)\)

**SOLUTION:**
\((6 - 8i)(9 + 2i) = 6(9) + 6(2i) - 8i(9) - 8i(2i)
= 54 + 12i - 72i - 16i^2
= 54 + 12i - 72i + 16(-1)
= 54 + 12i - 72i + 16
= 70 - 60i\)

**ANSWER:**
\(70 - 60i\)

14. \((3 + 2i)(-2 + 4i)\)

**SOLUTION:**
\((3 + 2i)(-2 + 4i) = 3(-2) + 3(4i) + 2i(-2) + 2i(4i)
= -6 + 12i - 4i + 8i^2
= -6 + 12i - 4i + 8(-1)
= -6 + 12i - 4i - 8
= -14 + 8i\)

**ANSWER:**
\(-14 + 8i\)

15. \(\frac{3-i}{4+2i}\)

**SOLUTION:**
\(\frac{3-i}{4+2i} = \frac{3-i}{4+2i} \cdot \frac{4-2i}{4-2i}
= \frac{(3-i)(4-2i)}{(4+2i)(4-2i)}
= \frac{12 - 6i - 4i + 2i^2}{16 - 4i^2}
= \frac{12 - 6i - 4i - 2}{16 - 4(-1)}
= \frac{10 - 10i}{20}
= \frac{10(1-i)}{2 \cdot 10}
= \frac{1-i}{2}
= \frac{1}{2} - \frac{1}{2}i\)

**ANSWER:**
\(\frac{1}{2} - \frac{1}{2}i\)
16. \( \frac{2 + i}{5 + 6i} \)

**SOLUTION:**

\[
\frac{2 + i}{5 + 6i} = \frac{2 + i}{5 + 6i} \cdot \frac{5 - 6i}{5 - 6i} = \frac{(2 + i)(5 - 6i)}{(5 + 6i)(5 - 6i)} = \frac{10 - 12i + 5i - 6i^2}{25 - 36i^2} = \frac{10 - 12i + 5i - 6(-1)}{25 - 36(-1)} = \frac{10 - 12i + 5i + 6}{25 + 36} = \frac{10}{61} - \frac{7}{61}i
\]

**ANSWER:**

\[
\frac{16}{61} - \frac{7}{61}i
\]

17. **ELECTRICITY** The current in one part of a series circuit is \( 5 - 3i \) amps. The current in another part of the circuit is \( 7 + 9i \) amps. Add these complex numbers to find the total current in the circuit.

**SOLUTION:**

Total current = \( (5 - 3j) + (7 + 9j) \)

\[= 5 - 3j + 7 + 9j = 12 + 6j \text{ amps} \]

**ANSWER:**

\( 12 + 6j \) amps

18. \( \sqrt{-121} \)

**SOLUTION:**

\[\sqrt{-121} = \sqrt{-1 \cdot 11 \cdot 11} = 11i \]

**ANSWER:**

\( 11i \)

19. \( \sqrt{-169} \)

**SOLUTION:**

\[\sqrt{-169} = \sqrt{-1 \cdot 13 \cdot 13} = \sqrt{-1} \cdot \sqrt{13^2} = 13i \]

**ANSWER:**

\( 13i \)

20. \( \sqrt{-100} \)

**SOLUTION:**

\[\sqrt{-100} = \sqrt{-1 \cdot 10 \cdot 10} = \sqrt{-1} \cdot \sqrt{10^2} = 10i \]

**ANSWER:**

\( 10i \)
4-4 Complex Numbers

21. \( \sqrt{-81} \)

**SOLUTION:**
\[
\sqrt{-81} = \sqrt{-1 \cdot 9 \cdot 9} \\
= \sqrt{-1} \cdot \sqrt{9^2} \\
= 9i
\]

**ANSWER:**
9i

22. \((-3i)(-7i)(2i)\)

**SOLUTION:**
\[
(-3i)(-7i)(2i) = (-3 \cdot -7 \cdot 2)(i \cdot i \cdot i) \\
= (-3 \cdot -7 \cdot 2)(-1 \cdot i) \\
= 42i
\]

**ANSWER:**
42i

23. \(4i(-6i)^2\)

**SOLUTION:**
\[
4i(-6i)^2 = (4i)(36i^2) \\
= (-144)(i) \\
= -144i
\]

**ANSWER:**
-144i

24. \(i^{11}\)

**SOLUTION:**
\[
i^{11} = i^{10} \cdot i \\
= (i^2)^5 \cdot i \\
= -1 \cdot i \\
= -i
\]

**ANSWER:**
-i

25. \(i^{25}\)

**SOLUTION:**
\[
i^{25} = i^{24} \cdot i \\
= (i^4)^6 \cdot i \\
= 1 \cdot i \\
= i
\]

**ANSWER:**
i

26. \((10 - 7i) + (6 + 9i)\)

**SOLUTION:**
\[
(10 - 7i) + (6 + 9i) = (10 + 6) + (-7i + 9i) \\
= 16 + 2i
\]

**ANSWER:**
16 + 2i
4-4 Complex Numbers

27. \((-3 + i) + (-4 - i)\)

**SOLUTION:**
\[ (-3 + i) + (-4 - i) = (-3 - 4) + (i - i) \]
\[ = -7 \]

**ANSWER:**
\(-7\)

28. \((12 + 5i) - (9 - 2i)\)

**SOLUTION:**
\[ (12 + 5i) - (9 - 2i) = 12 + 5i - 9 + 2i \]
\[ = 3 + 7i \]

**ANSWER:**
\(3 + 7i\)

29. \((11 - 8i) - (2 - 8i)\)

**SOLUTION:**
\[ (11 - 8i) - (2 - 8i) = 11 - 8i - 2 + 8i \]
\[ = 9 \]

**ANSWER:**
\(9\)

30. \((1 + 2i)(1 - 2i)\)

**SOLUTION:**
\[ (1 + 2i)(1 - 2i) = 1(1) + i(-2i) + 2i(1) + 2i(-2i) \]
\[ = 1 - 2i + 2i - 4i^2 \]
\[ = 1 - 2i + 2i - 4(-1) \]
\[ = 1 + 4 \]
\[ = 5 \]

**ANSWER:**
\(5\)

31. \((3 + 5i)(5 - 3i)\)

**SOLUTION:**
\[ (3 + 5i)(5 - 3i) = 3(5) + 3(-3i) + 5i(5) + 5i(-3i) \]
\[ = 15 - 9i + 25i - 15i^2 \]
\[ = 15 - 9i + 25i + 15 \]
\[ = 30 + 16i \]

**ANSWER:**
\(30 + 16i\)

32. \((4 - i)(6 - 6i)\)

**SOLUTION:**
\[ (4 - i)(6 - 6i) = 4(6) + i(6) - i(6) - i(-6i) \]
\[ = 24 + 6i - 6i + 6i^2 \]
\[ = 24 - 24i - 6i - 6 \]
\[ = 18 - 30i \]

**ANSWER:**
\(18 - 30i\)
Simplify.

\[ \frac{2i}{1+i} \]

**SOLUTION:**

\[
\frac{2i}{1+i} = \frac{2i(1-i)}{(1+i)(1-i)} = \frac{2i-2i^2}{1-i^2} = \frac{2i+2}{1+1} = \frac{2i+2}{2} = 1+i
\]

**ANSWER:**

1 + i

\[ \frac{5}{2+4i} \]

**SOLUTION:**

\[
\frac{5}{2+4i} = \frac{5(2-4i)}{2+4i(2-4i)} = \frac{10-20i}{10-16i^2} = \frac{10-20i}{10+16} = \frac{10-20i}{26} = \frac{1}{2} - i
\]

**ANSWER:**

\[ \frac{1}{2} - i \]

\[ \frac{5+i}{3i} \]

**SOLUTION:**

\[
\frac{5+i}{3i} = \frac{5+i}{3i} \cdot \frac{3i}{3i} = \frac{3i(5+i)}{9i^2} = \frac{15i+3i^2}{9i^2} = \frac{15i+3(-1)}{9(-1)} = \frac{15i-3}{-9} = -\frac{1}{3} - \frac{5}{3}i
\]

**ANSWER:**

\[ \frac{1}{3} - \frac{5}{3}i \]

Solve each equation.

36. \( 4x^2 + 4 = 0 \)

**SOLUTION:**

\( 4x^2 + 4 = 0 \)

\( 4x^2 = -4 \)

\( x^2 = -1 \)

\( x = \pm \sqrt{-1} \)

\( x = \pm i \)

**ANSWER:**

\( \pm i \)
4-4 Complex Numbers

37. $3x^2 + 48 = 0$

**SOLUTION:**

$3x^2 + 48 = 0$

$3x^2 = -48$

$x^2 = -16$

$x = \pm \sqrt{-16}$

$x = \pm 4i$

**ANSWER:**

$\pm 4i$

40. $6x^2 + 108 = 0$

**SOLUTION:**

$6x^2 + 108 = 0$

$6x^2 = -108$

$x^2 = -18$

$x = \pm \sqrt{-18}$

$x = \pm 3i\sqrt{2}$

**ANSWER:**

$\pm 3i\sqrt{2}$

38. $2x^2 + 50 = 0$

**SOLUTION:**

$2x^2 + 50 = 0$

$2x^2 = -50$

$x^2 = -25$

$x = \pm \sqrt{-25}$

$x = \pm 5i$

**ANSWER:**

$\pm 5i$

41. $8x^2 + 128 = 0$

**SOLUTION:**

$8x^2 + 128 = 0$

$8x^2 = -128$

$x^2 = -16$

$x = \pm \sqrt{-16}$

$x = \pm 4i$

**ANSWER:**

$\pm 4i$

39. $2x^2 + 10 = 0$

**SOLUTION:**

$2x^2 + 10 = 0$

$2x^2 = -10$

$x^2 = -5$

$x = \pm \sqrt{-5}$

$x = \pm i\sqrt{5}$

**ANSWER:**

$\pm i\sqrt{5}$
4-4 Complex Numbers

Find the values of x and y that make each equation true.

42. \(9 + 12i = 3x + 4yi\)

**SOLUTION:**
Set the real parts equal to each other.
\[9 = 3x\]
\[3 = x\]
Set the imaginary parts equal to each other.
\[12 = 4y\]
\[3 = y\]

**ANSWER:**
3, 3

43. \(x + 1 + 2yi = 3 - 6i\)

**SOLUTION:**
Set the real parts equal to each other.
\[x + 1 = 3\]
\[x = 3 - 1\]
\[x = 2\]
Set the imaginary parts equal to each other.
\[2y = -6\]
\[y = -3\]

**ANSWER:**
2, -3

44. \(2x + 7 + (3 - y)i = -4 + 6i\)

**SOLUTION:**
Set the real parts equal to each other.
\[2x + 7 = -4\]
\[2x + 7 - 7 = -4 - 7\]
\[2x = -11\]
\[x = \frac{-11}{2}\]
Set the imaginary parts equal to each other.
\[3 - y = 6\]
\[y = -3\]

**ANSWER:**
\[-\frac{11}{2}, -3\]

45. \(5 + y + (3x - 7)i = 9 - 3i\)

**SOLUTION:**
Set the real parts equal to each other.
\[5 + y = 9\]
\[y = 4\]
Set the imaginary parts equal to each other.
\[3x - 7 = -3\]
\[3x - 7 + 7 = -3 + 7\]
\[3x = 4\]
\[x = \frac{4}{3}\]

**ANSWER:**
\[\frac{4}{3}, 4\]
4-4 Complex Numbers

46. \( a + 3b + (3a - b)i = 6 + 6i \)

**SOLUTION:**

Set the real parts equal to each other.
\( a + 3b = 6 \rightarrow (1) \)

Set the imaginary parts equal to each other.
\( 3a - b = 6 \rightarrow (2) \)

Multiply the second equation by 3 and add the resulting equation to (1).

\[
\begin{align*}
9a - 3b &= 18 \quad (+) \\
10a &= 24 \\
a &= \frac{24}{10} \\
a &= \frac{12}{5}
\end{align*}
\]

Substitute \( a = \frac{12}{5} \) in (1).

\[
\begin{align*}
\frac{12}{5} + 3b &= 6 \\
12 + 15b &= 30 \\
15b &= 18 \\
b &= \frac{18}{15} \\
b &= \frac{6}{5}
\end{align*}
\]

**ANSWER:**

\[
\frac{12}{5}, \frac{6}{5}
\]

47. \( (2a - 4b)i + a + 5b = 15 + 58i \)

**SOLUTION:**

Set the real parts equal to each other.
\( a + 5b = 15 \rightarrow (1) \)

Set the imaginary parts equal to each other.
\( 2a - 4b = 58 \rightarrow (2) \)

Multiply the first equation by 2 and subtract the second equation from the resulting equation.

\[
\begin{align*}
2a + 10b &= 30 \\
2a - 4b &= 58 \\
14b &= -28 \\
b &= -2
\end{align*}
\]

Substitute \( b = -2 \) in (1).

\[
\begin{align*}
a + 5(-2) &= 15 \\
a - 10 &= 15 \\
a &= 25
\end{align*}
\]

**ANSWER:**

25, -2

48. \( \sqrt{-10} \cdot \sqrt{-24} \)

**SOLUTION:**

\[
\begin{align*}
\sqrt{-10} \cdot \sqrt{-24} &= \sqrt{-1 \cdot 2 \cdot 5} \cdot \sqrt{-1 \cdot 2 \cdot 2 \cdot 3} \\
&= \sqrt{1} \cdot \sqrt{2} \cdot \sqrt{5} \cdot \sqrt{-1} \cdot \sqrt{2} \cdot \sqrt{3} \\
&= i \cdot 2 \cdot \sqrt{15} \cdot i \cdot 2 \\
&= -4 \sqrt{15}
\end{align*}
\]

**ANSWER:**

\(-4 \sqrt{15}\)
4-4 Complex Numbers

49. \(4i \left(\frac{1}{2}i\right)^2 (-2i)^2\)

**SOLUTION:**

\[
4i \left(\frac{1}{2}i\right)^2 (-2i)^2 = 4i \left(\frac{1}{2}\right)^2 i^4 (-2)^2 i^2
= 4i \left(\frac{1}{4}\right) (-1)(4)(-1)
= 4i
\]

**ANSWER:**

4i

50. \(i^{41}\)

**SOLUTION:**

\[
i^{41} = i^{40} \cdot i
= (i^2)^{20} \cdot i
= 1 \cdot i
= i
\]

**ANSWER:**

i

52. \((8 - 5i) - (7 + i)\)

**SOLUTION:**

\[(8 - 5i) - (7 + i) = 8 - 5i - 7 - i = 1 - 6i\]

**ANSWER:**

1 - 6i

53. \((-6 - i)(3 - 3i)\)

**SOLUTION:**

\[(-6 - i)(3 - 3i) = -6(3) - 6(-3i) - i(3) - i(-3i)
= -18 + 18i - 3i - 3
= -21 + 15i\]

**ANSWER:**

-21 + 15i

51. \((4 - 6i) + (4 + 6i)\)

**SOLUTION:**

\[(4 - 6i) + (4 + 6i) = 4 + 4 - 6i + 6i
= 8\]

**ANSWER:**

8
54. \( \frac{(5 + i)^2}{3 - i} \)

SOLUTION:
\[
\frac{(5 + i)^2}{3 - i} = \frac{(5 + i)^2}{3 - i} \cdot \frac{3 + i}{3 + i} = \frac{(5 + i)^2 (3 + i)}{(3 - i)(3 + i)} = \frac{(25 - 1 + 10i)(3 + i)}{9 + 1} = \frac{(24 + 10i)(3 + i)}{10} = \frac{72 + 30i + 24i + 10i^2}{10} = \frac{72 + 30i + 24i - 10}{10} = \frac{62 + 54i}{10} = \frac{31 + 27i}{5 + 5i}
\]

ANSWER:
\[
\frac{31 + 27i}{5 + 5i}
\]

55. \( \frac{6 - i}{2 - 3i} \)

SOLUTION:
\[
\frac{6 - i}{2 - 3i} = \frac{6 - i}{2 - 3i} \cdot \frac{2 + 3i}{2 + 3i} = \frac{(6 - i)(2 + 3i)}{(2 - 3i)(2 + 3i)} = \frac{12 + 18i - 2i - 3i^2}{4 + 9} = \frac{12 + 18i - 2i + 3}{13} = \frac{15 + 16i}{13} = \frac{15}{13} + \frac{16}{13}i
\]

ANSWER:
\[
\frac{15}{13} + \frac{16}{13}i
\]

56. \( (-4 + 6i)(2 - i)(3 + 7i) \)

SOLUTION:
\[
(-4 + 6i)(2 - i)(3 + 7i) = (-4(2) - 4(-i) + 6i(2) + 5i(-i))(3 + 7i) = (-8 + 4i + 12i + 6)(3 + 7i) = (-2 + 16i)(3 + 7i) = -2(3) - 2(7i) + 16i(3) + 16i(7i) = -6 - 14i + 48i - 112 = -118 + 34i
\]

ANSWER:
\[-118 + 34i\]
57. \((1 + i)(2 + 3i)(4 - 3i)\)

**SOLUTION:**

\((1 + i)(2 + 3i)(4 - 3i) = (1(2) + 1(3i) + i(2) + i(3i))(4 - 3i) = (2 + 3i + 2i - 3i)(4 - 3i) = (-1 + 5i)(4 - 3i) = -4 + 3i + 20i + 15 = 11 + 23i\)

**ANSWER:**

11 + 23i

58. \(\frac{4 - i\sqrt{2}}{4 + i\sqrt{2}}\)

**SOLUTION:**

\[
\frac{4 - i\sqrt{2}}{4 + i\sqrt{2}} = \frac{4 - i\sqrt{2}}{4 + i\sqrt{2}} \cdot \frac{4 - i\sqrt{2}}{4 - i\sqrt{2}} = \frac{(4 - i\sqrt{2})(4 - i\sqrt{2})}{(4 + i\sqrt{2})(4 - i\sqrt{2})} = \frac{16 - 2 - 8i\sqrt{2}}{16 + 2} = \frac{16 - 2 - 8i\sqrt{2}}{18} = \frac{7 - 4i\sqrt{2}}{9}
\]

**ANSWER:**

\(\frac{7}{9} - \frac{4i\sqrt{2}}{9}\)

59. \(\frac{2 - i\sqrt{3}}{2 + i\sqrt{3}}\)

**SOLUTION:**

\[
\frac{2 - i\sqrt{3}}{2 + i\sqrt{3}} = \frac{2 - i\sqrt{3}}{2 + i\sqrt{3}} \cdot \frac{2 - i\sqrt{3}}{2 - i\sqrt{3}} = \frac{(2 - i\sqrt{3})(2 - i\sqrt{3})}{(2 + i\sqrt{3})(2 - i\sqrt{3})} = \frac{4 - 3 - 4i\sqrt{3}}{4 + 3} = \frac{1 - 4i\sqrt{3}}{7} = \frac{1}{7} - \frac{4i\sqrt{3}}{7}
\]

**ANSWER:**

\(\frac{1}{7} - \frac{4i\sqrt{3}}{7}\)

60. **ELECTRICITY** The impedance in one part of a series circuit is 7 + 8\(i\) ohms, and the impedance in another part of the circuit is 13 − 4\(i\) ohms. Add these complex numbers to find the total impedance in the circuit.

**SOLUTION:**

Total impedance = 7 + 8\(i\) + 13 − 4\(i\) = 20 + 4\(i\) ohms

**ANSWER:**

20 + 4\(i\) ohms
ELECTRICITY Use the formula \( V = C \cdot I \).

61. The current in a circuit is \( 3 + 6j \) amps, and the impedance is \( 5 - j \) ohms. What is the voltage?

**SOLUTION:**
We know that voltage can be calculated by
\[ V = C \cdot I. \]
\( V = \) Voltage
\( C = \) current
\( I = \) impedance
\[ V = (3 + 6j)(5 - j) \]
\[ = 15 - 3j + 30j + 6 \]
\[ = 21 + 27j \]
Therefore, the voltage is \( 21 + 27j \) Volts.

**ANSWER:**
\( 21 + 27j \) Volts

62. The voltage in a circuit is \( 20 - 12j \) volts, and the impedance is \( 6 - 4j \) ohms. What is the current?

**SOLUTION:**
We know that voltage can be calculated by
\[ V = C \cdot I. \]
\( V = \) Voltage
\( C = \) current
\( I = \) impedance
\[ 20 - 12j = I(6 - 4j) \]
\[ I = \frac{20 - 12j}{6 - 4j} \]
\[ = \frac{20 - 12j}{6 - 4j} \cdot \frac{6 + 4j}{6 + 4j} \]
\[ = \frac{(20 - 12j)(6 + 4j)}{(6 - 4j)(6 + 4j)} \]
\[ = \frac{120 + 80j - 72j + 48}{36 + 16} \]
\[ = \frac{168 + 8j}{52} \]
\[ = \frac{42}{13} + \frac{2}{13}j \]
Therefore, the current is \( \frac{42}{13} + \frac{2}{13}j \) Amps.

**ANSWER:**
\( \frac{42}{13} + \frac{2}{13}j \) Amps

63. Find the sum of \( ix^2 - (4 + 5i)x + 7 \) and \( 3x^2 + (2 + 6i) \)

**SOLUTION:**
\[ ix^2 - (4 + 5i)x + 7 + 3x^2 + (2 + 6i) \]
\[ = (3 + i)x^2 - 5ix - 4x + 2x + 6ix + 7 - 8i \]
\[ = (3 + i)x^2 + (-2 + i)x + 7 - 8i \]

**ANSWER:**
\( (3 + i)x^2 + (-2 + i)x - 8i + 7 \)
64. Simplify \([2 + i)x^2 - ix + 5 + i] - [(-3 + 4i)x^2 + (5 - 5i)x - 6].

**SOLUTION:**
\[
[(2 + i)x^2 - ix + 5 + i] - [(-3 + 4i)x^2 + (5 - 5i)x - 6] \\
= [(2 + i)x^2 - ix + 5 + i] - (-3 + 4i)x^2 - (5 - 5i)x + 6 \\
= 2x^2 + ix^2 - ix + 5 + i + 3x^2 - 4ix^2 - 5x + 5ix + 6 \\
= 5x^2 - 3ix^2 + i - 5x + 4ix + 11 \\
= (5 - 3i)x^2 + (-5 + 4i)x + i + 11
\]

**ANSWER:**
\[
(5 - 3i)x^2 + (-5 + 4i)x + i + 11
\]

65. **MULTIPLE REPRESENTATIONS** In this problem, you will explore quadratic equations that have complex roots. Use a graphing calculator.

a. **Algebraic** Write a quadratic equation in standard form with 3i and -3i as its roots.

b. **Graphical** Graph the quadratic equation found in part a by graphing its related function.

c. **Algebraic** Write a quadratic equation in standard form with 2 + i and 2 - i as its roots.

d. **Graphical** Graph the related function of the quadratic equation you found in part c. Use the graph to find the roots if possible. Explain.

e. **Analytical** How do you know when a quadratic equation will have only complex solutions?

**SOLUTION:**

a. Sample answer: \(x^2 + 9 = 0\)

d. Sample answer: A quadratic equation will have only complex solutions when the graph of the related function has no x-intercepts.

**ANSWER:**

a. Sample answer: \(x^2 + 9 = 0\)

d. Sample answer: \(x^2 - 4x + 5 = 0\)
4-4 Complex Numbers

67. **CHALLENGE** Simplify \((1 + 2i)^3\).

**SOLUTION:**

\[
\begin{align*}
(1 + 2i)^3 &= (1 + 2i)(1 + 2i)(1 + 2i) \\
&= (1 - 4i)(1 + 2i) \\
&= (-3 + 4i)(1 + 2i) \\
&= -3 - 6i + 4i - 8 \\
&= -11 - 2i
\end{align*}
\]

**ANSWER:**

\(-11 - 2i\)

68. **REASONING** Determine whether the following statement is always, sometimes, or never true. Explain your reasoning.

*Every complex number has both a real part and an imaginary part.*

**SOLUTION:**

Sample answer: Always. The value of 5 can be represented by \(5 + 0i\), and the value of \(3i\) can be represented by \(0 + 3i\).

**ANSWER:**

Sample answer: Always. The value of 5 can be represented by \(5 + 0i\), and the value of \(3i\) can be represented by \(0 + 3i\).

69. **OPEN ENDED** Write two complex numbers with a product of 20.

**SOLUTION:**

Sample answer: \((4 + 2i)(4 - 2i)\)

**ANSWER:**

Sample answer: \((4 + 2i)(4 - 2i)\)

e. Sample answer: A quadratic equation will have only complex solutions when the graph of the related function has no \(x\)-intercepts.

66. **CCSS CRITIQUE** Joe and Sue are simplifying \((2i)(3i)(4i)\). Is either of them correct? Explain your reasoning.

**Joe**

\[24i^3 = -24\]

**SOLUTION:**

Joe; \(i^3 = -i\), not \(-1\).

**ANSWER:**

Joe; \(i^3 = -i\), not \(-1\).

**Sue**

\[24i^3 = -24i\]

**SOLUTION:**

Sue; \(i^3 = -i\), not \(-1\).

**ANSWER:**

Sue; \(i^3 = -i\), not \(-1\).
4-4 Complex Numbers

70. **WRITING IN MATH** Explain how complex numbers are related to quadratic equations.

*SOLUTION:*
Some quadratic equations have complex solutions and cannot be solved using only the real numbers.

*ANSWER:*
Some quadratic equations have complex solutions and cannot be solved using only the real numbers.

71. **EXTENDED RESPONSE** Refer to the figure to answer the following.

![Diagram](image)

a. Name two congruent triangles with vertices in correct order.

b. Explain why the triangles are congruent.

c. What is the length of \( EC \)? Explain your procedure.

*SOLUTION:*

a. \( \triangle CBE \cong \triangle ADE \)

b. \( \angle AED \cong \angle CEB \) (Vertical angles)
\( DE \cong BE \) (Both have length \( x \.).
\( \angle ADE \cong \angle CBE \) (Given)
Consecutive angles and the included side are all congruent, so the triangles are congruent by the ASA Property.

c. \( EC \cong EA \) by CPCTC (corresponding parts of congruent triangles are congruent.) \( EA = 7 \), so \( EC = 7 \).

*ANSWER:*

a. \( \triangle CBE \cong \triangle ADE \)

b. \( \angle AED \cong \angle CEB \) (Vertical angles)
\( DE \cong BE \) (Both have length \( x \.).
\( \angle ADE \cong \angle CBE \) (Given) Consecutive angles and the included side are all congruent, so the triangles are congruent by the ASA Property.

c. \( EC \cong EA \) by CPCTC (corresponding parts of congruent triangles are congruent.) \( EA = 7 \), so \( EC = 7 \).
4-4 Complex Numbers

72. \((3 + 6)^2 = \)

A \(2 \times 3 + 2 \times 6\)

B \(9^2\)

C \(3^2 + 6^2\)

D \(3^2 \times 6^2\)

**SOLUTION:**
\[(3 + 6)^2 = 9^2\]

So, the correct option is B.

**ANSWER:**
B

73. SAT/ACT A store charges $49 for a pair of pants. This price is 40% more than the amount it costs the store to buy the pants. After a sale, any employee is allowed to purchase any remaining pairs of pants at 30% off the store’s cost. How much would it cost an employee to purchase the pants after the sale?

F $10.50

G $12.50

H $13.72

J $24.50

K $35.00

**SOLUTION:**
Let \(x\) be the original amount of the pants.
\[49 = 40\%x + x\]
\[49 = 0.4x + x\]
\[49 = 1.4x\]
\[x = \$35\]
\[35 \cdot \frac{30}{100} = \$10.50\]
\[35 - 10.50 = \$24.50\]

So, the correct option is J.

**ANSWER:**
J
4-4 Complex Numbers

74. What are the values of \(x\) and \(y\) when \((5 + 4i) - (x + yi) = (-1 - 3i)\)?

**SOLUTION:**
Set the real parts equal to each other.
\[5 - x = -1\]
\[x = 6\]

Set the imaginary parts equal to each other.
\[4 - y = -3\]
\[y = 7\]

So, the correct option is A.

**ANSWER:**
A

---

Solve each equation by factoring.

75. \(2x^2 + 7x = 15\)

**SOLUTION:**
Write the equation with right side equal to zero.
\[2x^2 + 7x - 15 = 0\]

Find factors of \(2(-15) = -30\) whose sum is 7.
\[10(-3) = -30\] and \(10 + (-3) = 7\)
\[2x^2 + 10x - 3x - 15 = 0\]
\[2x(x + 5) - 3(x + 5) = 0\]
\[2(x + 5)(2x - 3) = 0\]
\[x + 5 = 0\] or \(2x - 3 = 0\)
\[x = -5\] or \(x = \frac{3}{2}\)

Therefore, the roots are \(-5\) and \(\frac{3}{2}\).

**ANSWER:**
\[-5, \frac{3}{2}\]
76. $4x^2 - 12 = 22x$

**SOLUTION:**
Write the equation with right side equal to zero.
$4x^2 - 22x - 12 = 0$
Find factors of $4(-12) = -48$ whose sum is $-22$.
$-24(2) = -48$ and $2 + (-24) = -22$
$4x^2 - 24x + 2x - 12 = 0$
$4x(x - 6) + 2(x - 6) = 0$
$(x - 6)(4x + 2) = 0$
$\Rightarrow x - 6 = 0$ or $4x + 2 = 0$
$\Rightarrow x = 6$ or $x = -\frac{1}{2}$
Therefore, the roots are $-\frac{1}{2}$ and 6.

**ANSWER:**
$-\frac{1}{2}, 6$

77. $6x^2 = 5x + 4$

**SOLUTION:**
Write the equation with right side equal to zero.
$6x^2 - 5x - 4 = 0$
Find factors of $6(-4) = -24$ whose sum is $-5$.
$-8(3) = -24$ and $3 + (-8) = -5$
$6x^2 - 8x + 3x - 4 = 0$
$2x(3x - 4) + 1(3x - 4) = 0$
$(2x + 1)(3x - 4) = 0$
$\Rightarrow 2x + 1 = 0$ or $3x - 4 = 0$
$\Rightarrow x = -\frac{1}{2}$ or $x = \frac{4}{3}$
Therefore, the roots are $-\frac{1}{2}$ and $\frac{4}{3}$.

**ANSWER:**
$-\frac{1}{2}, \frac{4}{3}$

**NUMBER THEORY** Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

78. Their sum is $-3$, and their product is $-40$.

**SOLUTION:**
The quadratic equation to find the two real numbers with a sum of $-3$ and a product of $-40$ is $x^2 - 3x - 40 = 0$.
Solve the equation.
The two real numbers are 5 and $-8$.

**ANSWER:**
5, $-8$

79. Their sum is 19, and their product is 48.

**SOLUTION:**
The quadratic equation to find the two real numbers with a sum of 19 and a product of 48 is $x^2 + 19x + 48 = 0$.
Solve the equation.
The two real numbers are 3 and 16.

**ANSWER:**
3, 16

80. Their sum is $-15$, and their product is 56.

**SOLUTION:**
The quadratic equation to find the two real numbers with a sum of $-15$ and a product of 56 is $x^2 - 15x + 56 = 0$.
Solve the equation.
The two real numbers are $-7$ and $-8$.

**ANSWER:**
$-7$, $-8$
4-4 Complex Numbers

81. Their sum is –21, and their product is 108.

**SOLUTION:**
The quadratic equation to find the two real numbers with a sum of –21 and a product of 108 is
\[ x^2 – 21x + 108 = 0. \]
Solve the equation.
The two real numbers are –9 and –12.

**ANSWER:**
–9, –12

82. **RECREATION** Refer to the table.

a. Write a matrix that represents the cost of admission for residents and a matrix that represents the cost of admission for nonresidents.

b. Write the matrix that represents the additional cost for nonresidents.

c. Write a matrix that represents the difference in cost if a child or adult goes after 6:00 P.M. instead of before 6:00 P.M.

<table>
<thead>
<tr>
<th>Daily Admission Fees</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Residents</strong></td>
</tr>
<tr>
<td><strong>Time of day</strong></td>
</tr>
<tr>
<td>Before 6:00 P.M.</td>
</tr>
<tr>
<td>After 6:00 P.M.</td>
</tr>
<tr>
<td><strong>Nonresidents</strong></td>
</tr>
<tr>
<td><strong>Time of day</strong></td>
</tr>
<tr>
<td>Before 6:00 P.M.</td>
</tr>
<tr>
<td>After 6:00 P.M.</td>
</tr>
</tbody>
</table>

**SOLUTION:**

a.

\[
\begin{bmatrix}
3.00 & 4.50 \\
2.00 & 3.50
\end{bmatrix}
\]

b.

\[
\begin{bmatrix}
4.50 & 6.75 \\
3.00 & 5.25
\end{bmatrix}
\]

\[
\begin{bmatrix}
1.50 & 2.25 \\
1.00 & 1.75
\end{bmatrix}
\]

\[
\begin{bmatrix}
1.00 & 1.00 \\
1.50 & 1.50
\end{bmatrix}
\]

\[
\begin{bmatrix}
1.00 & 1.00 \\
1.50 & 1.50
\end{bmatrix}
\]
83. **PART-TIME JOBS** Terrell makes $10 an hour cutting grass and $12 an hour for raking leaves. He cannot work more than 15 hours per week. Graph two inequalities that Terrell can use to determine how many hours he needs to work at each job if he wants to earn at least $120 per week.

**SOLUTION:**
Let $x$ be the hours spent cutting grass and $y$ be the hours spent raking leaves. Terrell earns $10 per hour cutting grass and $12 per hour for raking leaves. He cannot work more than 15 hours per week and he wants to earn at least $120 per week.
Write an inequality that represents the hours Terrell can work.
$x + y \leq 15$
Write an inequality that represents his earnings for a week.
$10x + 12y \geq 120$

Graph the related equations and shade in the solution to the system of inequalities.

**Answer:**

Determine whether each trinomial is a perfect square trinomial. Write yes or no.

84. $x^2 + 16x + 64$

**SOLUTION:**
$x^2 + 16x + 64$ can be written as $(x + 8)^2$.
So, $x^2 + 16x + 64$ is a perfect square trinomial. The answer is “yes”.

**Answer:**
yes

85. $x^2 - 12x + 36$

**SOLUTION:**
$x^2 - 12x + 36$ can be written as $(x - 6)^2$.
So, $x^2 - 12x + 36$ is a perfect square trinomial. The answer is “yes”.

**Answer:**
yes

86. $x^2 + 8x - 16$

**SOLUTION:**
We cannot write the given trinomial as the perfect square format. So, the answer is “no”.

**Answer:**
no
4-4 Complex Numbers

87. \( x^2 - 14x - 49 \)

\[ \text{SOLUTION:} \]
We cannot write the given trinomial as the perfect square format. So, the answer is “no”.

\[ \text{ANSWER:} \]
no

88. \( x^2 + x + 0.25 \)

\[ \text{SOLUTION:} \]
\( x^2 + x + 0.25 \) can be written as \((x + 0.5)^2\).
So, \( x^2 + x + 0.25 \) is a perfect square trinomial. The answer is “yes”.

\[ \text{ANSWER:} \]
yes

89. \( x^2 + 5x + 6.25 \)

\[ \text{SOLUTION:} \]
\( x^2 + 5x + 6.25 \) can be written as \((x + 2.5)^2\).
So, \( x^2 + 5x + 6.25 \) is a perfect square trinomial. The answer is “yes”.

\[ \text{ANSWER:} \]
yes