

5-1 Operations with Polynomials

Simplify. Assume that no variable equals 0.

1. $(2a^3b^{-2})(-4a^2b^4)$

SOLUTION:

$$\begin{aligned}(2a^3b^{-2})(-4a^2b^4) &= -8a^{3+2}b^{-2+4} && \text{Product of Powers} \\ &= -8a^5b^2 && \text{Simplify.}\end{aligned}$$

ANSWER:

$$-8a^5b^2$$

2. $\frac{12x^4y^2}{2xy^5}$

SOLUTION:

$$\begin{aligned}\frac{12x^4y^2}{2xy^5} &= 6x^{4-1}y^{2-5} && \text{Quotient of Powers} \\ &= 6x^3y^{-3} && \text{Simplify.} \\ &= \frac{6x^3}{y^3} && \text{Simplify.}\end{aligned}$$

ANSWER:

$$\frac{6x^3}{y^3}$$

3. $\left(\frac{2a^2}{3b}\right)^3$

SOLUTION:

$$\begin{aligned}\left(\frac{2a^2}{3b}\right)^3 &= \frac{(2a^2)^3}{(3b)^3} && \text{Power of a Power} \\ &= \frac{2^3a^{2(3)}}{3^3b^3} && \text{Simplify.} \\ &= \frac{8a^6}{27b^3} && \text{Simplify.}\end{aligned}$$

ANSWER:

$$\frac{8a^6}{27b^3}$$

4. $(6g^5h^{-4})^3$

SOLUTION:

$$\begin{aligned}(6g^5h^{-4})^3 &= 6^3(g^5)^3(h^{-4})^3 && \text{Power of a Power} \\ &= 216g^{5(3)}h^{-4(3)} && \text{Simplify.} \\ &= 216g^{15}h^{-12} && \text{Simplify.} \\ &= \frac{216g^{15}}{h^{12}} && \text{Simplify.}\end{aligned}$$

ANSWER:

$$\frac{216g^{15}}{h^{12}}$$

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

5. $3x + 4y$

SOLUTION:

The expression is a polynomial because each term is a monomial. The degree of the first term is 1, and the degree of the second term is 1. The degree of the polynomial is 1.

ANSWER:

yes, 1

6. $\frac{1}{2}x^2 - 7y$

SOLUTION:

The expression is a polynomial because each term is a monomial. The degree of the first term is 2, and the degree of the second term is 1. The degree of the polynomial is 2.

ANSWER:

yes, 2

7. $x^2 + \sqrt{x}$

SOLUTION:

The expression is not a polynomial because \sqrt{x} is not a monomial.

ANSWER:

no

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8. $\frac{ab^3 - 1}{az^4 + 3}$

SOLUTION:

The expression is not a polynomial because $(az^4 + 3)^{-1}$ is not polynomial. A polynomial cannot contain variables in a denominator.

ANSWER:

no

Simplify.

9. $(x^2 - 5x + 2) - (3x^2 + x - 1)$

SOLUTION:

$$\begin{aligned} & (x^2 - 5x + 2) - (3x^2 + x - 1) \\ &= x^2 - 5x + 2 - 3x^2 - x + 1 \quad \text{Distributive Property} \\ &= -2x^2 - 6x + 3 \quad \text{Simplify.} \end{aligned}$$

ANSWER:

$$-2x^2 - 6x + 3$$

10. $(3a + 4b) + (6a - 6b)$

SOLUTION:

$$\begin{aligned} & (3a + 4b) + (6a - 6b) \\ &= 3a + 4b + 6a - 6b \quad \text{Remove grouping symbols.} \\ &= 9a - 2b \quad \text{Simplify.} \end{aligned}$$

ANSWER:

$$9a - 2b$$

11. $2a(4b + 5)$

SOLUTION:

$$\begin{aligned} 2a(4b + 5) &= 2a \cdot 4b + 2a \cdot 5 \quad \text{Distributive Property} \\ &= 8ab + 10a \quad \text{Simplify.} \end{aligned}$$

ANSWER:

$$8ab + 10a$$

12. $3x^2(2xy - 3xy^2 + 4x^2y^3)$

SOLUTION:

$$\begin{aligned} & 3x^2(2xy - 3xy^2 + 4x^2y^3) \\ &= 3x^2 \cdot 2xy - 3x^2 \cdot 3xy^2 + 3x^2 \cdot 4x^2y^3 \quad \text{Distributive Property} \\ &= 6x^3y - 9x^3y^2 + 12x^4y^3 \quad \text{Simplify.} \end{aligned}$$

ANSWER:

$$6x^3y - 9x^3y^2 + 12x^4y^3$$

13. $(n - 9)(n + 7)$

SOLUTION:

$$\begin{aligned} & (n - 9)(n + 7) \\ &= n(n + 7) - 9(n + 7) \quad \text{Distributive Property} \\ &= n \cdot n + 7 \cdot n - 9 \cdot n - 9 \cdot 7 \quad \text{FOIL} \\ &= n^2 + 7n - 9n - 63 \quad \text{Simplify.} \\ &= n^2 - 2n - 63 \quad \text{Combine like terms.} \end{aligned}$$

ANSWER:

$$n^2 - 2n - 63$$

14. $(a + 4)(a - 6)$

SOLUTION:

$$\begin{aligned} & (a + 4)(a - 6) \\ &= a(a - 6) + 4(a - 6) \quad \text{Distributive Property} \\ &= a \cdot a - 6 \cdot a + 4 \cdot a + 4(-6) \quad \text{FOIL} \\ &= a^2 - 6a + 4a - 24 \quad \text{Simplify.} \\ &= a^2 - 2a - 24 \quad \text{Combine like terms.} \end{aligned}$$

ANSWER:

$$a^2 - 2a - 24$$

15. **EXERCISE** Tara exercises 75 minutes a day. She does cardio, which burns an average of 10 Calories a minute, and weight training, which burns an average of 7.5 Calories a minute. Write a polynomial to represent the amount of Calories Tara burns in one day if she does x minutes of weight training.

SOLUTION:

Let x represent the minutes of weight training. Then, Tara does $75 - x$ minutes of cardio.

$$\begin{aligned} 7.5x + 10(75 - x) &= 7.5x + 750 - 10x \\ &= 750 - 2.5x \end{aligned}$$

ANSWER:

$$750 - 2.5x$$

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Simplify. Assume that no variable equals 0.

16. $(5x^3y^{-5})(4xy^3)$

SOLUTION:

$$\begin{aligned} (5x^3y^{-5})(4xy^3) &= 20x^{3+1}y^{-5+3} && \text{Product of Powers} \\ &= 20x^4y^{-2} && \text{Simplify.} \\ &= \frac{20x^4}{y^2} && \text{Simplify.} \end{aligned}$$

ANSWER:

$$\frac{20x^4}{y^2}$$

17. $(-2b^3c)(4b^2c^2)$

SOLUTION:

$$\begin{aligned} (-2b^3c)(4b^2c^2) \\ &= -8b^{3+2}c^{1+2} && \text{Product of Powers} \\ &= -8b^5c^3 && \text{Simplify.} \end{aligned}$$

ANSWER:

$$-8b^5c^3$$

18. $\frac{a^3n^7}{an^4}$

SOLUTION:

$$\begin{aligned} \frac{a^3n^7}{an^4} &= a^{3-1}n^{7-4} && \text{Quotient of Powers} \\ &= a^2n^3 && \text{Simplify.} \end{aligned}$$

ANSWER:

$$a^2n^3$$

19. $\frac{-y^3z^5}{y^2z^3}$

SOLUTION:

$$\begin{aligned} \frac{-y^3z^5}{y^2z^3} &= -y^{3-2}z^{5-3} && \text{Quotient of Powers} \\ &= -yz^2 && \text{Simplify.} \end{aligned}$$

ANSWER:

$$-yz^2$$

20. $\frac{-7x^5y^5z^4}{21x^7y^5z^2}$

SOLUTION:

$$\begin{aligned} \frac{-7x^5y^5z^4}{21x^7y^5z^2} \\ &= -\frac{1}{3}x^{5-7}y^{5-5}z^{4-2} && \text{Quotient of Powers} \\ &= -\frac{1}{3}x^{-2}y^0z^2 && \text{Simplify.} \\ &= -\frac{z^2}{3x^2} && \text{Simplify.} \end{aligned}$$

ANSWER:

$$\frac{z^2}{-3x^2}$$

21. $\frac{9a^7b^5c^5}{18a^5b^9c^3}$

SOLUTION:

$$\begin{aligned} \frac{9a^7b^5c^5}{18a^5b^9c^3} &= \frac{1}{2}a^{7-5}b^{5-9}c^{5-3} && \text{Quotient of Powers} \\ &= \frac{1}{2}a^2b^{-4}c^2 && \text{Simplify.} \\ &= \frac{a^2c^2}{2b^4} && \text{Simplify.} \end{aligned}$$

ANSWER:

$$\frac{a^2c^2}{2b^4}$$

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22. $(n^5)^4$

SOLUTION:

$$\begin{aligned}(n^5)^4 &= n^{5(4)} && \text{Power of a Power} \\ &= n^{20} && \text{Simplify.}\end{aligned}$$

ANSWER:

$$n^{20}$$

23. $(z^3)^6$

SOLUTION:

$$\begin{aligned}(z^3)^6 &= z^{3(6)} && \text{Power of a Power} \\ &= z^{18} && \text{Simplify.}\end{aligned}$$

ANSWER:

$$z^{18}$$

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

24. $2x^2 - 3x + 5$

SOLUTION:

The expression is a polynomial because each term is a monomial. The degree of the polynomial is 2.

ANSWER:

yes; 2

25. $a^3 - 11$

SOLUTION:

The expression is a polynomial because each term is a monomial. The degree of the polynomial is 3.

ANSWER:

yes; 3

26. $\frac{5np}{n^2} - \frac{2g}{h}$

SOLUTION:

The expression is not a polynomial because n^{-2} and h^{-1} are not monomials. Monomials cannot contain variables in the denominator.

ANSWER:

no

27. $\sqrt{m-7}$

SOLUTION:

The expression is not a polynomial because $\sqrt{m-7}$ is not a polynomial.

ANSWER:

no

CCSS REGULARITY Simplify.

28. $(6a^2 + 5a + 10) - (4a^2 + 6a + 12)$

SOLUTION:

$$\begin{aligned}&(6a^2 + 5a + 10) - (4a^2 + 6a + 12) \\ &= 6a^2 + 5a + 10 - 4a^2 - 6a - 12 && \text{Distributive Property} \\ &= 2a^2 - a - 2 && \text{Simplify.}\end{aligned}$$

ANSWER:

$$2a^2 - a - 2$$

29. $(7b^2 + 6b - 7) - (4b^2 - 2)$

SOLUTION:

$$\begin{aligned}&(7b^2 + 6b - 7) - (4b^2 - 2) \\ &= 7b^2 + 6b - 7 - 4b^2 + 2 && \text{Distributive Property} \\ &= 3b^2 + 6b - 5 && \text{Simplify.}\end{aligned}$$

ANSWER:

$$3b^2 + 6b - 5$$

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30. $3p(np - z)$

SOLUTION:

$$\begin{aligned} 3p(np - z) &= 3p \cdot np + 3p(-z) && \text{Distributive Property} \\ &= 3np^2 - 3pz && \text{Simplify.} \end{aligned}$$

ANSWER:

$$3np^2 - 3pz$$

31. $4x(2x^2 + y)$

SOLUTION:

$$\begin{aligned} 4x(2x^2 + y) &= 4x \cdot 2x^2 + 4x \cdot y && \text{Distributive Property} \\ &= 8x^3 + 4xy && \text{Simplify.} \end{aligned}$$

ANSWER:

$$8x^3 + 4xy$$

32. $(x - y)(x^2 + 2xy + y^2)$

SOLUTION:

$$\begin{aligned} &(x - y)(x^2 + 2xy + y^2) \\ &= x(x^2 + 2xy + y^2) - y(x^2 + 2xy + y^2) && \text{Distributive Property} \\ &= x^3 + 2x^2y + xy^2 - x^2y - 2xy^2 - y^3 && \text{Distributive Property} \\ &= x^3 + x^2y - xy^2 - y^3 && \text{Simplify.} \end{aligned}$$

ANSWER:

$$x^3 + x^2y - xy^2 - y^3$$

33. $(a + b)(a^3 - 3ab - b^2)$

SOLUTION:

$$\begin{aligned} &(a + b)(a^3 - 3ab - b^2) \\ &= a(a^3 - 3ab - b^2) + b(a^3 - 3ab - b^2) && \text{Distributive Property} \\ &= a^4 - 3a^2b - ab^2 + a^3b - 3ab^2 - b^3 && \text{Distributive Property} \\ &= a^4 + a^3b - 3a^2b - 4ab^2 - b^3 && \text{Simplify.} \end{aligned}$$

ANSWER:

$$a^4 + a^3b - 3a^2b - 4ab^2 - b^3$$

34. $4(a^2 + 5a - 6) - 3(2a^3 + 4a - 5)$

SOLUTION:

$$\begin{aligned} &4(a^2 + 5a - 6) - 3(2a^3 + 4a - 5) = \\ &= (4a^2 + 20a - 24) - (6a^3 + 12a - 15) && \text{Distributive Property} \\ &= 4a^2 + 20a - 24 - 6a^3 - 12a + 15 && \text{Distributive Property} \\ &= -6a^3 + 4a^2 + 8a - 9 && \text{Simplify.} \end{aligned}$$

ANSWER:

$$-6a^3 + 4a^2 + 8a - 9$$

35. $5c(2c^2 - 3c + 4) + 2c(7c - 8)$

SOLUTION:

$$\begin{aligned} &5c(2c^2 - 3c + 4) + 2c(7c - 8) \\ &= (10c^3 - 15c^2 + 20c) + (14c^2 - 16c) && \text{Distributive Property} \\ &= 10c^3 - 15c^2 + 20c + 14c^2 - 16c && \text{Remove grouping symbols.} \\ &= 10c^3 - c^2 + 4c && \text{Simplify.} \end{aligned}$$

ANSWER:

$$10c^3 - c^2 + 4c$$

36. $5xy(2x - y) + 6y^2(x^2 + 6)$

SOLUTION:

$$\begin{aligned} &5xy(2x - y) + 6y^2(x^2 + 6) \\ &= 5xy(2x) - 5xy(y) + 6y^2(x^2) + 6y^2(6) && \text{Distributive Property} \\ &= 10x^2y - 5xy^2 + 6x^2y^2 + 36y^2 && \text{Simplify.} \end{aligned}$$

ANSWER:

$$10x^2y - 5xy^2 + 6x^2y^2 + 36y^2$$

37. $3ab(4a - 5b) + 4b^2(2a^2 + 1)$

SOLUTION:

$$\begin{aligned} &3ab(4a - 5b) + 4b^2(2a^2 + 1) \\ &= 3ab(4a) - 3ab(5b) + 4b^2(2a^2) + 4b^2(1) && \text{Distributive Property} \\ &= 12a^2b - 15ab^2 + 8a^2b^2 + 4b^2 && \text{Simplify.} \end{aligned}$$

ANSWER:

$$12a^2b + 8a^2b^2 - 15ab^2 + 4b^2$$

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38. $(x - y)(x + y)(2x + y)$

SOLUTION:

$$\begin{aligned} & (x - y)(x + y)(2x + y) \\ &= (x(x + y) - y(x + y))(2x + y) && \text{Factor out GCF.} \\ &= (x^2 + xy - xy - y^2)(2x + y) && \text{Distributive Property} \\ &= x^2(2x + y) + xy(2x + y) - xy(2x + y) - y^2(2x + y) && \text{Distributive Property} \\ &= 2x^3 + x^2y + 2x^2y + xy^2 - 2x^2y - xy^2 - 2xy^2 - y^3 && \text{Distributive Property} \\ &= 2x^3 + x^2y - 2xy^2 - y^3 && \text{Simplify.} \end{aligned}$$

ANSWER:

$$2x^3 + x^2y - 2xy^2 - y^3$$

39. $(a + b)(2a + 3b)(2x - y)$

SOLUTION:

$$\begin{aligned} & (a + b)(2a + 3b)(2x - y) \\ &= (a(2a + 3b) + b(2a + 3b))(2x - y) && \text{Factor out GCF.} \\ &= (2a^2 + 3ab + 2ab + 3b^2)(2x - y) && \text{Distributive Property} \\ &= 2a^2(2x - y) + 3ab(2x - y) + 2ab(2x - y) + 3b^2(2x - y) && \text{Distributive Property} \\ &= 4a^2x - 2a^2y + 6abx - 3aby + 4abx - 2aby + 6b^2x - 3b^2y && \text{Distributive Property} \\ &= 4a^2x - 2a^2y + 10abx - 5aby + 6b^2x - 3b^2y && \text{Simplify.} \end{aligned}$$

ANSWER:

$$4a^2x - 2a^2y + 10abx - 5aby + 6b^2x - 3b^2y$$

40. **PAINTING** Connor has hired two painters to paint his house. The first painter charges \$12 an hour and the second painter charges \$11 an hour. It will take 15 hours of labor to paint the house.

a. Write a polynomial to represent the total cost of the job if the first painter does x hours of the labor.

b. Write a polynomial to represent the total cost of the job if the second painter does y hours of the labor.

SOLUTION:

a. If the first painter does x hours of the labor, then the second painter does $15 - x$ hours of the labor. Then the total cost of the job is $12(x) + 11(15 - x)$. Simplify this expression.

$$\begin{aligned} 12(x) + 11(15 - x) &= 12x + 165 - 11x \\ &= x + 165 \end{aligned}$$

b. If the second painter does y hours of the labor, then the first painter does $15 - y$ hours of the labor. Then the total cost of the job is $12(15 - y) + 11(y)$. Simplify this expression.

$$\begin{aligned} 12(15 - y) + 11(y) &= 180 - 12y + 11y \\ &= 180 - y \end{aligned}$$

ANSWER:

a. $x + 165$

b. $180 - y$

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Simplify. Assume that no variable equals 0.

$$41. \left(\frac{8x^2y^3}{24x^3y^2} \right)^4$$

SOLUTION:

$$\begin{aligned} & \left(\frac{8x^2y^3}{24x^3y^2} \right)^4 \\ &= \left(\frac{1}{3}x^{2-3}y^{3-2} \right)^4 && \text{Quotient of Powers} \\ &= \left(\frac{1}{3}x^{-1}y^1 \right)^4 && \text{Simplify.} \\ &= \left(\frac{y}{3x} \right)^4 && \text{Definition of negative exponents} \\ &= \frac{y^4}{(3x)^4} && \text{Simplify.} \\ &= \frac{y^4}{81x^4} && \text{Power of a Product} \end{aligned}$$

ANSWER:

$$\frac{y^4}{81x^4}$$

$$42. \left(\frac{12a^3b^5}{4a^6b^3} \right)^3$$

SOLUTION:

$$\begin{aligned} & \left(\frac{12a^3b^5}{4a^6b^3} \right)^3 \\ &= \left(3a^{3-6}b^{5-3} \right)^3 && \text{Quotient of Powers} \\ &= \left(3a^{-3}b^2 \right)^3 && \text{Simplify.} \\ &= \left(\frac{3b^2}{a^3} \right)^3 && \text{Definition of negative exponents} \\ &= \frac{(3b^2)^3}{(a^3)^3} && \text{Power of a Quotient} \\ &= \frac{27b^6}{a^9} && \text{Simplify.} \end{aligned}$$

ANSWER:

$$\frac{27b^6}{a^9}$$

$$43. \left(\frac{4x^{-2}y^3}{xy^{-4}} \right)^{-2}$$

SOLUTION:

$$\begin{aligned} & \left(\frac{4x^{-2}y^3}{xy^{-4}} \right)^{-2} \\ &= \left(4x^{-2-1}y^{3-(-4)} \right)^{-2} && \text{Quotient of Powers} \\ &= \left(\frac{4y^7}{x^3} \right)^{-2} && \text{Simplify.} \\ &= \left(\frac{x^3}{4y^7} \right)^2 && \text{Definition of negative exponents} \\ &= \frac{(x^3)^2}{(4y^7)^2} && \text{Power of a Quotient} \\ &= \frac{x^6}{16y^{14}} && \text{Power of a Power} \end{aligned}$$

ANSWER:

$$\frac{x^6}{16y^{14}}$$

$$44. \left(\frac{5a^{-7}b^2}{ab^{-6}} \right)^{-3}$$

SOLUTION:

$$\begin{aligned} & \left(\frac{5a^{-7}b^2}{ab^{-6}} \right)^{-3} \\ &= \left(5a^{-7-1}b^{2-(-6)} \right)^{-3} && \text{Quotient of Powers} \\ &= \left(\frac{5b^8}{a^8} \right)^{-3} && \text{Simplify.} \\ &= \left(\frac{a^8}{5b^8} \right)^3 && \text{Definition of negative exponent} \\ &= \frac{(a^8)^3}{(5b^8)^3} && \text{Power of a Quotient} \\ &= \frac{a^{24}}{125b^{24}} && \text{Power of a Power} \end{aligned}$$

ANSWER:

$$\frac{a^{24}}{125b^{24}}$$

5-1 Operations with Polynomials

45. $(a^2b^3)(ab)^{-2}$

SOLUTION:

$$\begin{aligned} & (a^2b^3)(ab)^{-2} \\ &= \frac{(a^2b^3)}{(ab)^2} && \text{Definition of negative exponent} \\ &= \frac{a^2b^3}{a^2b^2} && \text{Power of a Product} \\ &= a^{2-2}b^{3-2} && \text{Quotient of Powers} \\ &= b && \text{Simplify.} \end{aligned}$$

ANSWER:

b

46. $(-3x^3y)^2(4xy^2)$

SOLUTION:

$$\begin{aligned} & (-3x^3y)^2(4xy^2) \\ &= (9x^6y^2)(4xy^2) && \text{Power of a Product} \\ &= 36x^{6+1}y^{2+2} && \text{Simplify.} \\ &= 36x^7y^4 && \text{Simplify.} \end{aligned}$$

ANSWER:

$36x^7y^4$

47. $\frac{3c^2d(2c^3d^5)}{15c^4d^2}$

SOLUTION:

$$\begin{aligned} & \frac{3c^2d(2c^3d^5)}{15c^4d^2} \\ &= \frac{6c^{2+3}d^{1+5}}{15c^4d^2} && \text{Product of Powers} \\ &= \frac{6c^5d^6}{15c^4d^2} && \text{Simplify.} \\ &= \frac{2}{5}c^{5-4}d^{6-2} && \text{Definition of negative exponent} \\ &= \frac{2}{5}cd^4 && \text{Simplify.} \end{aligned}$$

ANSWER:

$\frac{2}{5}cd^4$

48. $\frac{-10g^6h^9(g^2h^3)}{30g^3h^3}$

SOLUTION:

$$\begin{aligned} & \frac{-10g^6h^9(g^2h^3)}{30g^3h^3} \\ &= \frac{-10g^8h^{12}}{30g^3h^3} && \text{Product of Powers} \\ &= -\frac{1}{3}g^{8-3}h^{12-3} && \text{Definition of negative exponent} \\ &= -\frac{1}{3}g^5h^9 && \text{Simplify.} \end{aligned}$$

ANSWER:

$-\frac{1}{3}g^5h^9$

49. $\frac{5x^4y^2(2x^5y^6)}{20x^3y^5}$

SOLUTION:

$$\begin{aligned} & \frac{5x^4y^2(2x^5y^6)}{20x^3y^5} = \frac{10x^9y^8}{20x^3y^5} && \text{Product of Powers} \\ &= \frac{1}{2}x^{9-3}y^{8-5} && \text{Definition of negative exponent} \\ &= \frac{1}{2}x^6y^3 && \text{Simplify.} \end{aligned}$$

ANSWER:

$\frac{1}{2}x^6y^3$

50. $\frac{-12n^7p^5(n^2p^4)}{36n^6p^7}$

SOLUTION:

$$\begin{aligned} & \frac{-12n^7p^5(n^2p^4)}{36n^6p^7} \\ &= \frac{-12n^9p^9}{36n^6p^7} && \text{Product of Powers} \\ &= -\frac{1}{3}n^{9-6}p^{9-7} && \text{Definition of negative exponent} \\ &= -\frac{1}{3}n^3p^2 && \text{Simplify.} \end{aligned}$$

ANSWER:

$-\frac{1}{3}n^3p^2$

5-1 Operations with Polynomials

51. **ASTRONOMY** The light from the Sun takes approximately 8 minutes to reach Earth. So if you are outside right now you are basking in sunlight that the Sun emitted approximately 8 minutes ago.

Light travels very fast, at a speed of about 3×10^8 meters per second. How long would it take light to get here from the Andromeda galaxy, which is approximately 2.367×10^{21} meters away?

- a. How long does it take light from Andromeda to reach Earth?
b. The average distance from the Sun to Mars is approximately 2.28×10^{11} meters. How long does it take light from the Sun to reach Mars?

SOLUTION:

a.

$$\begin{aligned}\text{Time taken} &= \frac{\text{Distance}}{\text{Speed}} \\ &= \frac{2.367 \times 10^{21}}{3 \times 10^8} \\ &= 7.89 \times 10^{12}\end{aligned}$$

Convert 7.89×10^{12} seconds to years. There are about 31,557,600 seconds in the average year.

$$\begin{aligned}\frac{7.89 \times 10^{12}}{31,557,600} \\ \approx 250,190.26\end{aligned}$$

Light takes about 250,000 years to reach Earth from Andromeda.

b.

$$\begin{aligned}\text{Time taken} &= \frac{\text{Distance}}{\text{Speed}} \\ &= \frac{2.28 \times 10^{11}}{3 \times 10^8} \\ &= 760\end{aligned}$$

Light takes 760 seconds or about 12.67 minutes to reach Mars from the Sun.

ANSWER:

- a. 7.89×10^{12} s or about 250,190.26 years
b. 760 seconds or about 12.67 min

Simplify.

$$52. \frac{1}{4}g^2(8g + 12h - 16gh^2)$$

SOLUTION:

$$\begin{aligned}&\frac{1}{4}g^2(8g + 12h - 16gh^2) \\ &= \frac{1}{4}g^2 \cdot 8g + \frac{1}{4}g^2 \cdot 12h - \frac{1}{4}g^2 \cdot 16gh^2 \quad \text{Distributive Property} \\ &= 2g^3 + 3g^2h - 4g^3h^2 \quad \text{Simplify.}\end{aligned}$$

ANSWER:

$$2g^3 + 3g^2h - 4g^3h^2$$

$$53. \frac{1}{3}n^3(6n - 9p + 18np^4)$$

SOLUTION:

$$\begin{aligned}&\frac{1}{3}n^3(6n - 9p + 18np^4) \\ &= \frac{1}{3}n^3 \cdot 6n - \frac{1}{3}n^3 \cdot 9p + \frac{1}{3}n^3 \cdot 18np^4 \quad \text{Distributive Property} \\ &= 2n^4 - 3n^3p + 6n^4p^4 \quad \text{Simplify.}\end{aligned}$$

ANSWER:

$$2n^4 - 3n^3p + 6n^4p^4$$

$$54. x^{-2}(x^4 - 3x^3 + x^{-1})$$

SOLUTION:

$$\begin{aligned}&x^{-2}(x^4 - 3x^3 + x^{-1}) \\ &= x^{-2} \cdot x^4 - 3x^{-2} \cdot x^3 + x^{-2} \cdot x^{-1} \quad \text{Distributive Property} \\ &= x^2 - 3x + x^{-3} \quad \text{Simplify.} \\ &= x^2 - 3x + \frac{1}{x^3} \quad \text{Simplify.}\end{aligned}$$

ANSWER:

$$x^2 - 3x + \frac{1}{x^3}$$

$$55. a^{-3}b^2(ba^3 + b^{-1}a^2 + b^{-2}a)$$

SOLUTION:

$$\begin{aligned}&a^{-3}b^2(ba^3 + b^{-1}a^2 + b^{-2}a) \\ &= a^{-3}b^2 \cdot ba^3 + a^{-3}b^2 \cdot b^{-1}a^2 + a^{-3}b^2 \cdot b^{-2}a \quad \text{Distributive Property} \\ &= b^3 + a^{-1}b + a^{-2} \quad \text{Simplify.} \\ &= b^3 + \frac{b}{a} + \frac{1}{a^2} \quad \text{Definition of negative exponent}\end{aligned}$$

ANSWER:

$$b^3 + \frac{b}{a} + \frac{1}{a^2}$$

5-1 Operations with Polynomials

56. $(g^3 - h)(g^3 + h)$

SOLUTION:

$$\begin{aligned} & (g^3 - h)(g^3 + h) \\ &= g^6 + g^3h - g^3h - h^2 \quad \text{Distributive Property} \\ &= g^6 - h^2 \quad \text{Combine like terms.} \end{aligned}$$

ANSWER:

$$g^6 - h^2$$

57. $(n^2 - 7)(2n^3 + 4)$

SOLUTION:

$$(n^2 - 7)(2n^3 + 4) = 2n^5 - 14n^3 + 4n^2 - 28 \quad \text{Distributive Property}$$

ANSWER:

$$2n^5 - 14n^3 + 4n^2 - 28$$

58. $(2x - 2y)^3$

SOLUTION:

$$\begin{aligned} & (2x - 2y)^3 \quad \text{Definition of exponent} \\ &= (2x - 2y)(2x - 2y)(2x - 2y) \\ &= (4x^2 - 8xy + 4y^2)(2x - 2y) \quad \text{Distributive Property} \\ &= 4x^2(2x - 2y) - 8xy(2x - 2y) + 4y^2(2x - 2y) \quad \text{Distributive Property} \\ &= 8x^3 - 8x^2y - 16x^2y + 16xy^2 + 8xy^2 - 8y^3 \quad \text{Distributive Property} \\ &= 8x^3 - 24x^2y + 24xy^2 - 8y^3 \quad \text{Combine like terms.} \end{aligned}$$

ANSWER:

$$8x^3 - 24x^2y + 24xy^2 - 8y^3$$

59. $(4n - 5)^3$

SOLUTION:

$$\begin{aligned} & (4n - 5)^3 \quad \text{Definition of exponent} \\ &= (4n - 5)(4n - 5)(4n - 5) \\ &= (16n^2 - 20n - 20n + 25)(4n - 5) \quad \text{Distributive Property} \\ &= 16n^2(4n - 5) - 20n(4n - 5) - 20n(4n - 5) + 25(4n - 5) \quad \text{Distributive Property} \\ &= 64n^3 - 80n^2 - 80n^2 + 100n - 80n^2 + 100n + 100n - 125 \quad \text{Distributive Property} \\ &= 64n^3 - 240n^2 + 300n - 125 \quad \text{Combine like terms.} \end{aligned}$$

ANSWER:

$$64n^3 - 240n^2 + 300n - 125$$

60. $(3z - 2)^3$

SOLUTION:

$$\begin{aligned} & (3z - 2)^3 \quad \text{Definition of exponent} \\ &= (3z - 2)(3z - 2)(3z - 2) \\ &= (9z^2 - 12z + 4)(3z - 2) \quad \text{Distributive Property} \\ &= 9z^2(3z - 2) - 12z(3z - 2) + 4(3z - 2) \quad \text{Distributive Property} \\ &= 27z^3 - 18z^2 - 36z^2 + 24z + 12z - 8 \quad \text{Distributive Property} \\ &= 27z^3 - 54z^2 + 36z - 8 \quad \text{Combine like terms.} \end{aligned}$$

ANSWER:

$$27z^3 - 54z^2 + 36z - 8$$

61. **CCSS MODELING** The polynomials $0.108x^2 - 0.876x + 474.1$ and $0.047x^2 + 9.694x + 361.7$ approximate the number of bachelor's degrees, in thousands, earned by males and females, respectively, where x is the number of years after 1971.

a. Find the polynomial that represents the total number of bachelor's degrees (in thousands) earned by both men and women.

b. Find the polynomial that represents the difference between bachelor's degrees earned by men and by women.

SOLUTION:

a. Total number of bachelor's degrees:

$$\begin{array}{r} 0.108x^2 - 0.876x + 474.1 \\ (+) 0.047x^2 + 9.694x + 361.7 \\ \hline 0.155x^2 + 8.818x + 835.8 \end{array}$$

b. Difference between bachelor's degrees earned by men and by women:

$$\begin{array}{r} 0.108x^2 - 0.876x + 474.1 \\ (-) 0.047x^2 + 9.694x + 361.7 \\ \hline 0.061x^2 - 10.57x + 112.4 \end{array}$$

ANSWER:

a. $0.155x^2 + 8.818x + 835.8$

b. $0.061x^2 - 10.57x + 112.4$

5-1 Operations with Polynomials

62. If $5^{k+7} = 5^{2k-3}$, what is the value of k ?

SOLUTION:

The bases are equal, so equate the powers.

$$\begin{aligned} k + 7 &= 2k - 3 \\ k - 2k &= -3 - 7 \\ -k &= -10 \\ k &= 10 \end{aligned}$$

ANSWER:

10

63. What value of k makes $q^{41} = q^{4k} \cdot q^5$ true?

SOLUTION:

$$\begin{aligned} q^{41} &= q^{4k} \cdot q^5 \\ q^{41} &= q^{4k+5} \end{aligned}$$

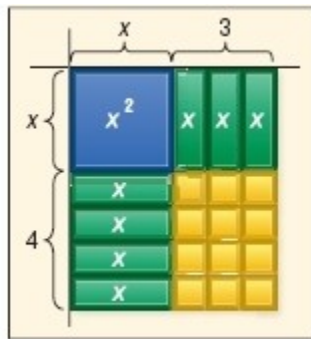
The bases are equal, so equate the powers.

$$\begin{aligned} 41 &= 4k + 5 \\ 41 - 5 &= 4k \\ 36 &= 4k \\ \frac{36}{4} &= k \\ 9 &= k \end{aligned}$$

ANSWER:

9

64. **MULTIPLE REPRESENTATIONS** Use the model that represents the product of $x + 3$ and $x + 4$.
- a. GEOMETRIC** The area of the each rectangle is the product of its length and width. Use the model to find the product of $x + 3$ and $x + 4$.
- b. ALGEBRAIC** Use FOIL to find the product of $x + 3$ and $x + 4$.
- c. VERBAL** Explain how each term of the product is represented in the model.



SOLUTION:

- a.** The model consists of a square with an area of x^2 , 7 rectangles each with area x , and 12 unit squares. So, the product of $x + 3$ and $x + 4$ is $x^2 + 7x + 12$.

- b.** Find the product of $x + 3$ and $x + 4$ using the FOIL method.

$$\begin{aligned} (x + 3)(x + 4) &= x^2 + 4x + 3x + 12 \\ &= x^2 + 7x + 12 \end{aligned}$$

- c.** Each term is represented by one or more rectangles with an area that represents the variable and the power in the term.

ANSWER:

a. $x^2 + 7x + 12$

b. $x^2 + 7x + 12$

- c.** Each term is represented by one or more rectangles with an area that represents the variable and the power in the term.

5-1 Operations with Polynomials

65. **PROOF** Show how the property of negative exponents can be proven using the Quotient of Powers Property and the Zero Power Property.

SOLUTION:

To prove the property of negative exponents show that

$$\begin{aligned}\frac{1}{a^n} &= a^{-n} \\ \frac{1}{a^n} &= \frac{a^0}{a^n} && 1 = a^0 \\ &= a^{0-n} && \text{Quotient of Powers} \\ &= a^{-n} && \text{Simplify.}\end{aligned}$$

ANSWER:

$$\frac{1}{a^n} = \frac{a^0}{a^n} = a^{0-n} = a^{-n}$$

66. **CHALLENGE** What happens to the quantity of x^{-y} increases, for $y > 0$ and $x \geq 1$?

SOLUTION:

Make a table of values.

x	y	x^{-y}
2	1	$2^{-1} = \frac{1}{2}$
3	2	$3^{-2} = \frac{1}{9}$
4	3	$4^{-3} = \frac{1}{64}$
5	4	$5^{-4} = \frac{1}{625}$
6	5	$6^{-5} = \frac{1}{7776}$
7	6	$7^{-6} = \frac{1}{117,649}$
8	7	$8^{-7} = \frac{1}{2,097,152}$
9	8	$9^{-8} = \frac{1}{43,046,721}$
10	9	$10^{-9} = \frac{1}{1,000,000,000}$
11	10	$11^{-10} = \frac{1}{25,937,424,601}$

Sample answer: It approaches 0.

ANSWER:

Sample answer: It approaches 0.

67. **REASONING** Explain why the expression 0^{-2} is undefined.

SOLUTION:

Sample answer: Following the property of negative exponents, $0^{-2} = \frac{1}{0^2}$ which is undefined. There would be a 0 in the denominator, which makes the expression undefined.

ANSWER:

Sample answer: We would have a 0 in the denominator, which makes the expression undefined.

68. **OPEN ENDED** Write three different expressions that are equivalent to x^{12} .

SOLUTION:

Sample answer:

$$x^9 \cdot x^3 \quad \text{Product of Powers}$$

$$\frac{x^{14}}{x^2} \quad \text{Quotient of Powers}$$

$$(x^6)^2 \quad \text{Power of a Power}$$

ANSWER:

Sample answer:

$$x^9 \cdot x^3, \frac{x^{14}}{x^2}, (x^6)^2$$

5-1 Operations with Polynomials

69. **WRITING IN MATH** Explain why properties of exponents are useful in astronomy. Include an explanation of how to find the amount of time it takes for light from a source to reach a planet.

SOLUTION:

Sample answer: Astronomy deals with very large numbers that are sometimes difficult to work with because they contain so many digits. Properties of exponents make very large or very small numbers more manageable. As long as you know how far away a planet is from a light source you can divide that distance by the speed of light to obtain how long it will take light to reach that planet.

ANSWER:

Sample answer: Astronomy deals with very large numbers that are sometimes difficult to work with because they contain so many digits. Properties of exponents make very large or very small numbers more manageable. As long as you know how far away a planet is from a light source you can divide that distance by the speed of light to obtain how long it will take light to reach that planet.

70. **Simplify** $\frac{(2x^2)^3}{12x^4}$.

SOLUTION:

$$\begin{aligned}\frac{(2x^2)^3}{12x^4} &= \frac{2^3 x^{2(3)}}{12x^4} && \text{Power of a Power} \\ &= \frac{8x^6}{12x^4} && \text{Simplify.} \\ &= \frac{2}{3} x^{6-4} && \text{Quotient of Powers} \\ &= \frac{2x^2}{3} && \text{Simplify.}\end{aligned}$$

ANSWER:

$$\frac{2x^2}{3}$$

71. **STATISTICS** For the numbers a , b , and c , the average (arithmetic mean) is twice the median. If $a = 0$ and $a < b < c$, what is the value of $\frac{c}{b}$?

A 2

B 3

C 4

D 5

SOLUTION:

$$\frac{a+b+c}{3} = 2b$$

Substitute 0 for a and simplify.

$$\begin{aligned}\frac{0+b+c}{3} &= 2b \\ b+c &= 6b \\ c &= 5b \\ \frac{c}{b} &= 5\end{aligned}$$

D is the correct option.

ANSWER:

D

72. Which is not a factor of $x^3 - x^2 - 2x$?

F x

G $x + 1$

H $x - 1$

J $x - 2$

SOLUTION:

$$\begin{aligned}x^3 - x^2 - 2x &= x(x^2 - x - 2) \\ &= x(x-2)(x+1)\end{aligned}$$

Thus, in the given option $x - 1$ is not a factor of $x^3 - x^2 - 2x$.

So, H is the correct option.

ANSWER:

H

5-1 Operations with Polynomials

73. **SAT/ACT** The expression $(-6 + i)^2$ is equivalent to which of the following expressions?

A 35
 B $-12i$
 C $-12 + i$
 D $35 - 12i$
 E $37 - 12i$

SOLUTION:

$$\begin{aligned} (-6 + i)^2 &= (-6 + i)(-6 + i) \\ &= 36 - 6i - 6i + i^2 \\ &= 36 - 12i + i^2 \\ &= 36 - 12i - 1 \\ &= 35 - 12i \end{aligned}$$

The correct choice is D.

ANSWER:

D

Solve each inequality algebraically.

74. $x^2 - 6x \leq 16$

SOLUTION:

First, write the related equation and factor it.

$$\begin{aligned} x^2 - 6x &= 16 \\ x^2 - 6x - 16 &= 0 \\ (x - 8)(x + 2) &= 0 \\ x - 8 = 0 \text{ or } x + 2 = 0 \\ x = 8 \text{ or } x = -2 \end{aligned}$$

The two numbers divide the number line into three regions, $x \leq -2$, $-2 \leq x \leq 8$ and $x \geq 8$. Test a value from each interval to see if it satisfies the original inequality.

$x \leq -2$	$-2 \leq x \leq 8$	$x \geq 8$
Test $x = -5$	Test $x = 1$	Test $x = 10$
$x^2 - 6x \leq 16$	$x^2 - 6x \leq 16$	$x^2 - 6x \leq 16$
$(-5)^2 - 6(-5) \stackrel{?}{\leq} 16$	$1^2 - 6(1) \stackrel{?}{\leq} 16$	$10^2 - 6(10) \stackrel{?}{\leq} 16$
$55 \not\leq 16$	$-5 \leq 16$	$40 \not\leq 16$

Note that, the points $x = -5$ and $x = 10$ are not included in the solution. Therefore, the solution set is $\{x \mid -2 \leq x \leq 8\}$.

ANSWER:

$$-2 \leq x \leq 8$$

5-1 Operations with Polynomials

75. $x^2 + 3x > 40$

SOLUTION:

First, write the related equation and factor it.

$$\begin{aligned}x^2 + 3x - 40 &= 0 \\(x + 8)(x - 5) &= 0 \\x + 8 = 0 \text{ or } x - 5 &= 0 \\x = -8 \text{ or } x &= 5\end{aligned}$$

The two numbers divide the number line into three regions, $x < -8$, $-8 < x < 5$ and $x > 5$. Test a value from each interval to see if it satisfies the original inequality.

$x < -8$	$-8 < x < 5$	$x > 5$
Test $x = -10$	Test $x = 1$	Test $x = 10$
$x^2 + 3x > 40$	$x^2 + 3x > 40$	$x^2 + 3x > 40$
$(-10)^2 + 3(10) \overset{?}{>} 40$	$1^2 + 3(1) \overset{?}{>} 40$	$6^2 + 3(6) \overset{?}{>} 40$
$130 > 40$	$4 \not> 40$	$54 > 40$

Note that, the point $x = 1$ is not included in the solution. Therefore, the solution set is $\{x \mid x > 5 \text{ or } x < -8\}$.

ANSWER:

$x > 5 \text{ or } x < -8$

76. $2x^2 - 12 \leq -5x$

SOLUTION:

First, write the related equation and factor it.

$$\begin{aligned}2x^2 + 5x - 12 &= 0 && \text{Related equation} \\x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-12)}}{2(2)} &&& \text{Quadratic Formula: } a=2, b=5, c=-12 \\&= \frac{-5 \pm \sqrt{121}}{4} && \text{Simplify.} \\&= \frac{-5 \pm 11}{4} && \text{Simplify.} \\&= -4 \text{ or } 1.5 && \text{Simplify.}\end{aligned}$$

The two numbers divide the number line into three regions $x \leq -4$, $-4 \leq x \leq 1.5$ and $x \geq 1.5$. Test a value from each interval to see if it satisfies the original inequality.

$x \leq -4$	$-4 \leq x \leq 1.5$	$x \geq 1.5$
Test $x = -5$	Test $x = 1$	Test $x = 2$
$2x^2 - 12 \leq -5x$	$2x^2 - 12 \leq -5x$	$2x^2 - 12 \leq -5x$
$2(-5)^2 - 12 \overset{?}{\leq} -5(-5)$	$2(1)^2 - 12 \overset{?}{\leq} -5(1)$	$2(2)^2 - 12 \overset{?}{\leq} -5(2)$
$38 \not\leq 25$	$-10 \leq -5$	$-4 \not\leq -10$

Note that, the points $x = -5$ and $x = 2$ are not included in the solution. Therefore, the solution set is $\{x \mid -4 \leq x \leq 1.5\}$.

ANSWER:

$-4 \leq x \leq 1.5$

5-1 Operations with Polynomials

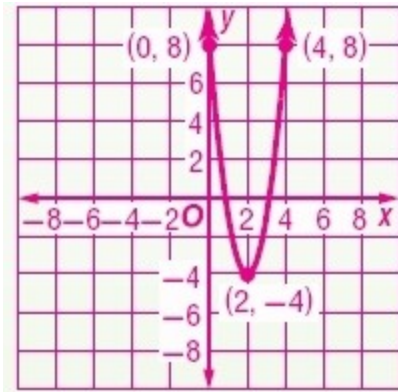
Graph each function.

77. $y = 3(x - 2)^2 - 4$

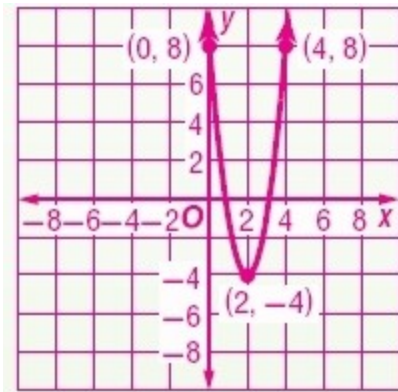
SOLUTION:

The vertex is at $(2, -4)$. The axis of symmetry is $x = 2$. Because $a = 3$, the graph opens up.

Graph of the function $y = 3(x - 2)^2 - 4$:



ANSWER:

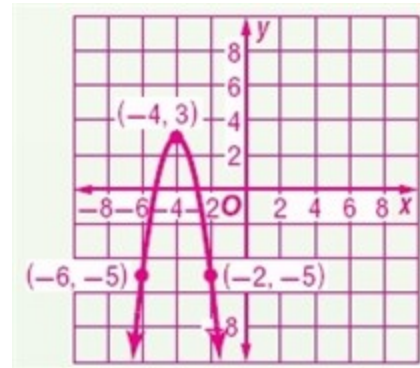


78. $y = -2(x + 4)^2 + 3$

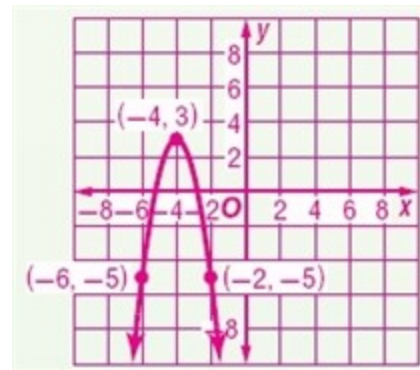
SOLUTION:

The vertex is at $(-4, 3)$. The axis of symmetry is $x = -4$. Because $a = -2$, the graph opens down.

Graph of the function $y = -2(x + 4)^2 + 3$.



ANSWER:



5-1 Operations with Polynomials

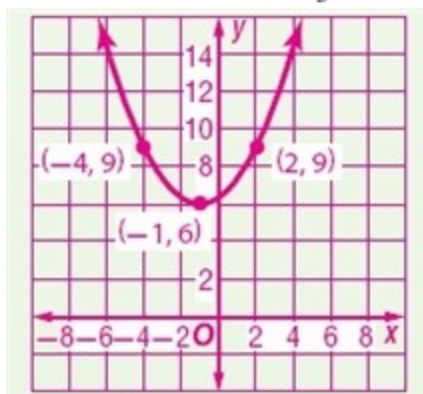
79. $y = \frac{1}{3}(x+1)^2 + 6$

SOLUTION:

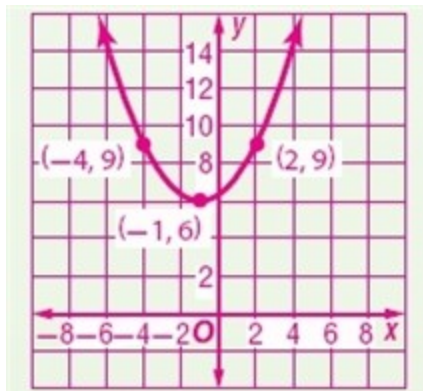
The vertex is at $(-1, 6)$. The axis of symmetry is $x =$

-1 . Because $a = \frac{1}{3}$, the graph opens up.

Graph of the function $y = \frac{1}{3}(x+1)^2 + 6$:



ANSWER:



80. **BASEBALL** A baseball player hits a high pop-up with an initial upward velocity of 30 meters per second, 1.4 meters above the ground. The height $h(t)$ of the ball in meters t seconds after being hit is modeled by $h(t) = -4.9t^2 + 30t + 1.4$. How long does an opposing player have to get under the ball if he catches it 1.7 meters above the ground? Does your answer seem reasonable? Explain.



SOLUTION:

Substitute 1.7 for $h(t)$ in the function $h(t) = -4.9t^2 + 30t + 1.4$.

$$-4.9t^2 + 30t + 1.4 = 1.7$$

$$-4.9t^2 + 30t - 0.3 = 0$$

Factor it.

$$\begin{aligned} & \frac{-30 \pm \sqrt{30^2 - 4(-4.9)(-0.3)}}{2(-4.9)} \\ &= \frac{-30 \pm \sqrt{894.12}}{-9.8} \\ &= \frac{-30 \pm 29.9}{-9.8} \\ &\approx 6.1 \text{ or } 0.01 \end{aligned}$$

Sample answer: About 6.1 seconds; this answer seems reasonable. The equation has two solutions. The first solution, 0.01 second, is the time required for the ball to rise from 1.4 m to 1.7 m, and 6.1 seconds is the time required for the ball to come back down to 1.7 m.

ANSWER:

Sample answer: About 6.1 seconds; this answer seems reasonable. The equation has two solutions. The first solution, 0.01 second, is the time required for the ball to rise from 1.4 m to 1.7 m, and 6.1 seconds is the time required for the ball to come back down to 1.7 m.

5-1 Operations with Polynomials

Evaluate each determinant.

$$81. \begin{vmatrix} 3 & 0 & -2 \\ -1 & 4 & 3 \\ 5 & -2 & -1 \end{vmatrix}$$

SOLUTION:

Rewrite the first two columns to the right of the determinant.

$$\begin{vmatrix} 3 & 0 & -2 \\ -1 & 4 & 3 \\ 5 & -2 & -1 \end{vmatrix} \begin{vmatrix} 3 & 0 \\ -1 & 4 \\ 5 & -2 \end{vmatrix}$$

Find the products of the elements of the diagonals.

$$\begin{aligned} 3(4)(-1) &= -12 & 5(4)(-2) &= -40 \\ 0(3)(5) &= 0 & -2(3)(3) &= -18 \\ -2(-1)(-2) &= -4 & -1(-1)(0) &= 0 \end{aligned}$$

Find the sum of each group.

$$\begin{aligned} -12 + 0 - 4 &= -16 \\ -40 - 18 + 0 &= -58 \end{aligned}$$

Subtract the sum of the second group from the sum of the first group.

$$-16 - (-58) = 42$$

The value of the determinant is 42.

ANSWER:

42

$$82. \begin{vmatrix} -2 & -4 & -6 \\ 0 & 6 & -5 \\ -1 & 3 & -1 \end{vmatrix}$$

SOLUTION:

Rewrite the first two columns to the right of the determinant.

$$\begin{vmatrix} -2 & -4 & -6 \\ 0 & 6 & -5 \\ -1 & 3 & -1 \end{vmatrix} \begin{vmatrix} -2 & -4 \\ 0 & 6 \\ -1 & 3 \end{vmatrix}$$

Find the products of the elements of the diagonals.

$$\begin{aligned} -2(6)(-1) &= 12 & -1(6)(-6) &= 36 \\ -4(-5)(-1) &= -20 & 3(-5)(-2) &= 30 \\ -6(0)(3) &= 0 & -1(0)(-4) &= 0 \end{aligned}$$

Find the sum of each group.

$$\begin{aligned} 12 + (-20) + 0 &= -8 \\ 36 + 30 + 0 &= 66 \end{aligned}$$

Subtract the sum of the second group from the sum of the first group.

$$-8 - 66 = -74$$

The value of the determinant is -74.

ANSWER:

-74

5-1 Operations with Polynomials

$$83. \begin{vmatrix} -3 & -1 & -2 \\ -2 & 3 & 4 \\ 6 & 1 & 0 \end{vmatrix}$$

SOLUTION:

Rewrite the first two columns to the right of the determinant.

$$\begin{vmatrix} -3 & -1 & -2 \\ -2 & 3 & 4 \\ 6 & 1 & 0 \end{vmatrix} \begin{vmatrix} -3 & -1 \\ -2 & 3 \\ 6 & 1 \end{vmatrix}$$

Find the products of the elements of the diagonals.

$$\begin{aligned} -3(3)(0) &= 0 & 6(3)(-2) &= -36 \\ -1(4)(6) &= -24 & 1(4)(-3) &= -12 \\ -2(-2)(1) &= 4 & 0(-2)(-1) &= 0 \end{aligned}$$

Find the sum of each group.

$$\begin{aligned} 0 + (-24) + 4 &= -20 \\ -36 + (-12) + 0 &= -48 \end{aligned}$$

Subtract the sum of the second group from the sum of the first group.

$$-20 - (-48) = 28$$

The value of the determinant is 28.

ANSWER:

28

84. **FINANCIAL LITERACY** A couple is planning to invest \$15,000 in certificates of deposit (CDs). For tax purposes, they want their total interest the first year to be \$800. They want to put \$1000 more in a 2-year CD than in a 1-year CD and then invest the rest in a 3-year CD. How much should they invest in each type of CD?

Years	1	2	3
Rate	3.4%	5.0%	6.0%

SOLUTION:

Let x represent the amount deposited in the first year.

So, the 2nd year deposit = $x + 1000$ and the 3rd year deposit = $14,000 - 2x$.

An equation that represents the situation is:

$$\begin{aligned} x(0.034) + (x + 1000)(0.05) + (14000 - 2x)(0.06) &= 800 \\ 0.034x + 0.05x + 50 + 840 - 0.12x &= 800 \\ -0.036x + 890 &= 800 \\ -0.036x &= -90 \\ x &= 2500 \end{aligned}$$

First year deposit = \$2500

Second year deposit = \$2500 + \$1000 = \$3500

Third year deposit = $14000 - 2(2500) = \$9000$

ANSWER:

\$2500 in the 1-year; \$3500 in the 2-year; \$9000 in the 3-year

5-1 Operations with Polynomials

Find the slope of the line that passes through each pair of points.

85. (6, -2) and (-2, -9)

SOLUTION:

Let (x_1, y_1) be (6, -2) and (x_2, y_2) be (-2, -9).

Substitute into the slope formula to find the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-9 - (-2)}{-2 - 6} \\ &= \frac{-7}{-8} \\ &= \frac{7}{8} \end{aligned}$$

ANSWER:

$$\frac{7}{8}$$

86. (-4, -1) and (3, 8)

SOLUTION:

Let (x_1, y_1) be (-4, -1) and (x_2, y_2) be (3, 8).

Substitute into the slope formula to find the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{8 - (-1)}{3 - (-4)} \\ &= \frac{9}{7} \end{aligned}$$

ANSWER:

$$\frac{9}{7}$$

87. (3, 0) and (-7, -5)

SOLUTION:

Let (x_1, y_1) be (3, 0) and (x_2, y_2) be (-7, -5).

Substitute into the slope formula to find the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-5 - 0}{-7 - 3} \\ &= \frac{-5}{-10} \\ &= \frac{1}{2} \end{aligned}$$

ANSWER:

$$\frac{1}{2}$$

88. $\left(\frac{1}{2}, \frac{2}{3}\right)$ and $\left(\frac{1}{4}, \frac{1}{3}\right)$

SOLUTION:

Let (x_1, y_1) be $\left(\frac{1}{2}, \frac{2}{3}\right)$ and (x_2, y_2) be $\left(\frac{1}{4}, \frac{1}{3}\right)$.

Substitute into the slope formula to find the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{\frac{1}{3} - \frac{2}{3}}{\frac{1}{4} - \frac{1}{2}} \\ &= \frac{-\frac{1}{3}}{-\frac{1}{4}} \\ &= \frac{4}{3} \end{aligned}$$

ANSWER:

$$\frac{4}{3}$$

5-1 Operations with Polynomials

89. $\left(\frac{2}{5}, \frac{1}{4}\right)$ and $\left(\frac{1}{10}, \frac{1}{12}\right)$

SOLUTION:

Let (x_1, y_1) be $\left(\frac{2}{5}, \frac{1}{4}\right)$ and (x_2, y_2) be $\left(\frac{1}{10}, \frac{1}{12}\right)$.

Substitute into the slope formula to find the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{\frac{1}{12} - \frac{1}{4}}{\frac{1}{10} - \frac{2}{5}} \\ &= \frac{-\frac{1}{6}}{-\frac{3}{10}} \\ &= \frac{5}{9} \end{aligned}$$

ANSWER:

$$\frac{5}{9}$$

90. $(-4.5, 2.5)$ and $(-3, -1)$

SOLUTION:

Let (x_1, y_1) be $(-4.5, 2.5)$ and (x_2, y_2) be $(-3, -1)$.

Substitute into the slope formula to find the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1 - 2.5}{-3 - (-4.5)} \\ &= \frac{-3.5}{1.5} \\ &= -\frac{7}{3} \end{aligned}$$

ANSWER:

$$-\frac{7}{3}$$

Factor each polynomial.

91. $12ax^3 + 20bx^2 + 32cx$

SOLUTION:

Factor out the GCF.

$$12ax^3 + 20bx^2 + 32cx = 4x(3ax^2 + 5bx + 8c)$$

ANSWER:

$$4x(3ax^2 + 5bx + 8c)$$

92. $x^2 + 2x + 6 + 3x$

SOLUTION:

Group terms such that there is a GCF in each group.

$$\begin{aligned} &x^2 + 2x + 3x + 6 \\ &= x(x + 2) + 3(x + 2) \quad \text{Group terms and factor the GCF from each group.} \\ &= (x + 3)(x + 2) \quad \text{Distributive Property} \end{aligned}$$

ANSWER:

$$(x + 3)(x + 2)$$

93. $12y^2 + 9y + 8y + 6$

SOLUTION:

$$\begin{aligned} &12y^2 + 9y + 8y + 6 \\ &= 3y(4y + 3) + 2(4y + 3) \quad \text{Group terms and factor the GCF from each group.} \\ &= (3y + 2)(4y + 3) \quad \text{Distributive Property} \end{aligned}$$

ANSWER:

$$(3y + 2)(4y + 3)$$

94. $2my + 7x + 7m + 2xy$

SOLUTION:

$$\begin{aligned} &2my + 7m + 2xy + 7x \\ &= m(2y + 7) + x(2y + 7) \quad \text{Group terms and factor the GCF from each group.} \\ &= (m + x)(2y + 7) \quad \text{Distributive Property} \end{aligned}$$

ANSWER:

$$(m + x)(2y + 7)$$

95. $8ax - 6x - 12a + 9$

SOLUTION:

$$\begin{aligned} &8ax - 6x - 12a + 9 \\ &= 2x(4a - 3) - 3(4a - 3) \quad \text{Group terms and factor the GCF from each group.} \\ &= (2x - 3)(4a - 3) \quad \text{Distributive Property} \end{aligned}$$

ANSWER:

$$(2x - 3)(4a - 3)$$

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96. $10x^2 - 14xy - 15x + 21y$

SOLUTION:

$$\begin{aligned} & 10x^2 - 14xy - 15x + 21y \\ &= 2x(5x - 7y) - 3(5x - 7y) \quad \text{Group terms and factor the GCF from each group.} \\ &= (2x - 3)(5x - 7y) \quad \text{Distributive Property} \end{aligned}$$

ANSWER:

$$(2x - 3)(5x - 7y)$$