6-1 Operations on Functions

Find \( f + g \)(x), \( f - g \)(x), \( f \cdot g \)(x), and \( \frac{f}{g} \)(x) for each \( f \)(x) and \( g \)(x). Indicate any restrictions in domain or range.

1. \( f(x) = x + 2 \)
\( g(x) = 3x - 1 \)

**ANSWER:**
\[ (f + g)(x) = 4x + 1 \]
\[ (f - g)(x) = -2x + 3 \]
\[ (f \cdot g)(x) = 3x^2 + 5x - 2 \]
\[ \left( \frac{f}{g} \right)(x) = \frac{x + 2}{3x - 1}, \ x \neq \frac{1}{3} \]

For each pair of functions, find \( f \circ g \) and \( g \circ f \), if they exist. State the domain and range for each composed function.

3. \( f = \{(2, 5), (6, 10), (12, 9), (7, 6)\} \)
\( g = \{(9, 11), (6, 15), (10, 13), (5, 8)\} \)

**ANSWER:**
\( f \circ g \) is undefined;
\( g \circ f = \{(2, 8), (6, 13), (12, 11), (7, 15)\} \)

Find \( [f \circ g](x) \) and \( [g \circ f](x) \), if they exist. State the domain and range for each composed function.

5. \( f(x) = -3x \)
\( g(x) = 5x - 6 \)

**ANSWER:**
\[ [f \circ g](x) = -15x + 18 \]
\[ [g \circ f](x) = -15x - 6 \]

7. **CCSS MODELING** Dora has 8% of her earnings deducted from her paycheck for a college savings plan. She can choose to take the deduction either before taxes are withheld, which reduces her taxable income, or after taxes are withheld. Dora’s tax rate is 17.5%. If her pay before taxes and deductions is $950, will she save more money if the deductions are taken before or after taxes are withheld? Explain.

**ANSWER:**
Either way, she will have $228.95 taken from her paycheck. If she takes the college savings plan deduction before taxes, $76 will go to her college plan and $152.95 will go to taxes. If she takes the college savings plan deduction after taxes, only $62.70 will go to her college plan and $166.25 will go to taxes.
6-1 Operations on Functions

11. \( f(x) = 3x \)
   \( g(x) = -2x + 6 \)

**ANSWER:**
\( (f + g)(x) = x + 6 \)
\( (f - g)(x) = 5x - 6 \)
\( (f \cdot g)(x) = -6x^2 + 18x \)
\( \left( \frac{f}{g} \right)(x) = \frac{3x}{-2x + 6}, x \neq 3 \)

13. \( f(x) = x^2 \)
   \( g(x) = x - 5 \)

**ANSWER:**
\( (f + g)(x) = x^2 + x - 5 \)
\( (f - g)(x) = x^2 - x + 5 \)
\( (f \cdot g)(x) = x^3 - 5x^2 \)
\( \left( \frac{f}{g} \right)(x) = \frac{x^2}{x - 5}, x \neq 5 \)

15. \( f(x) = 3x^2 - 4 \)
   \( g(x) = x^2 - 8x + 4 \)

**ANSWER:**
\( (f + g)(x) = 4x^2 - 8x \)
\( (f - g)(x) = 2x^2 + 8x - 8 \)
\( (f \cdot g)(x) = 3x^4 - 24x^3 + 32x^2 - 16 \)
\( \left( \frac{f}{g} \right)(x) = \frac{3x^2 - 4}{x^2 - 8x + 4}, x \neq 4 \pm 2\sqrt{3} \)

For each pair of functions, find \( f \circ g \) and \( g \circ f \). If they exist. State the domain and range for each composed function.

17. \( f = \{(-8, -4), (0, 4), (2, 6), (-6, -2)\} \)
   \( g = \{(4, -4), (-2, -1), (-4, 0), (6, -5)\} \)

**ANSWER:**
\( f \circ g = \{(-8, -4), (0, 4), (2, 6), (-6, -2)\} \)
\( g \circ f = \{(-8, -4), (-2, -1), (-4, 0), (6, -5)\} \)

19. \( f = \{(5, 13), (-4, -2), (-8, -11), (3, 1)\} \)
   \( g = \{(-8, 2), (-4, 1), (3, -3), (5, 7)\} \)

**ANSWER:**
\( f \circ g \) is undefined; \( g \circ f \) is undefined.

21. \( f = \{(-15, -5), (-4, 12), (1, 7), (3, 9)\} \)
   \( g = \{3, -9), (7, 2), (8, -6), (12, 0)\} \)

**ANSWER:**
\( f \circ g \) is undefined; \( g \circ f = \{(-4, 0), (1, 2)\} \).

23. \( f = \{7, -3\}, (-10, -3), (-7, -8), (-3, 6)\) \)
   \( g = \{4, -3), (3, -7), (9, 8), (-4, -4)\) \)

**ANSWER:**
\( f \circ g = \{(4, 6), (-3, 8)\} \); \( g \circ f \) is undefined.

25. \( f = \{(-4, -1), (-2, 6), (-1, 10), (4, 11)\} \)
   \( g = \{-1, 5), (3, -4), (6, 4), (10, 8)\} \)

**ANSWER:**
\( f \circ g = \{(3, -1), (6, 11)\} \)
\( g \circ f = \{(-4, 5), (-2, 4), (-1, 8)\} \)
6-1 Operations on Functions

Find \( f \circ g \) and \( g \circ f \) if they exist. State the domain and range for each composed function.

27. \( f(x) = 2x \)
   \( g(x) = x + 5 \)

   **ANSWER:**
   \[ (f \circ g)(x) = 2x + 10; \]
   \[ (g \circ f)(x) = 2x + 5 \]
   \( D = \{ \text{all real numbers} \}, \ R = \{ \text{all even numbers} \} \)
   \( D = \{ \text{all real numbers} \}, \ R = \{ \text{all odd numbers} \} \)

29. \( f(x) = x + 5 \)
   \( g(x) = 3x - 7 \)

   **ANSWER:**
   \[ (f \circ g)(x) = 3x - 2; \]
   \[ (g \circ f)(x) = 3x + 8 \]
   \( D = \{ \text{all real numbers} \}, \ R = \{ \text{all real numbers} \} \)
   \( D = \{ \text{all real numbers} \}, \ R = \{ \text{all real numbers} \} \)

31. \( f(x) = x^2 + 6x - 2 \)
   \( g(x) = x - 6 \)

   **ANSWER:**
   \[ (f \circ g)(x) = x^2 - 6x - 2; \]
   \[ (g \circ f)(x) = x^2 + 6x - 8 \]
   \( D = \{ \text{all real numbers} \}, \ R = \{ y \mid y \geq -11 \} \)
   \( D = \{ \text{all real numbers} \}, \ R = \{ y \mid y \geq -17 \} \)

33. \( f(x) = 4x - 1 \)
   \( g(x) = x^3 + 2 \)

   **ANSWER:**
   \[ (f \circ g)(x) = 4x^2 + 7; \]
   \[ (g \circ f)(x) = 4x^3 - 48x^2 + 12x + 1 \]
   \( D = \{ \text{all real numbers} \}, \ R = \{ \text{all real numbers} \} \)
   \( D = \{ \text{all real numbers} \}, \ R = \{ \text{all real numbers} \} \)
37. **CCSS SENSE-MAKING** Ms. Smith wants to buy an HDTV, which is on sale for 35% off the original price of $2299. The sales tax is 6.25%.

   a. Write two functions representing the price after the discount, \( p(x) \), and the price after sales tax, \( t(x) \).

   \[
p(x) = 0.65x; \quad t(x) = 1.0625x
   \]

   **ANSWER:**

   b. Which composition of functions represents the price of the HDTV, \( [p \circ t](x) \) or \( [t \circ p](x) \)? Explain your reasoning.

   **ANSWER:**

   c. How much will Ms. Smith pay for the HDTV?

   **ANSWER:**

43. \( h[f(−5)] \)

44. \( f[h(−3)] \)

45. \( f[g(3a)] \)

46. \( g[f(a^2−a)] \)

Perform each operation if \( f(x) = x^2 + x - 12 \) and \( g(x) = x - 3 \). State the domain of the resulting function.

**ANSWER:**

39. \( 2(\cdot f)(x) \)

**ANSWER:**

\[
2(\cdot f)(x) = 2x^3 - 4x^2 - 30x + 72; \quad D = \{ \text{all real numbers} \}
\]

If \( f(x) = 5x \), \( g(x) = -2x + 1 \), and \( h(x) = x^2 + 6x + 8 \), find each value.

41. \( f[g(−2)] \)

**ANSWER:**

25
6-1 Operations on Functions

51. **EMPLOYMENT** The number of women and men age 16 and over employed each year in the United States can be modeled by the following equations, where \( x \) is the number of years since 1994 and \( y \) is the number of people in thousands.

- women: \( y = 1086.4x + 56,610 \)
- men: \( y = 999.2x + 66,450 \)

a. Write a function that models the total number of men and women employed in the United States during this time.

b. If \( f \) is the function for the number of men, and \( g \) is the function for the number of women, what does \((f - g)(x)\) represent?

**ANSWER:**

a. \( y = 2085.6x + 123,060 \)

b. The function represents the difference in the number of men and women employed in the U.S.

If \( f(x) = x + 2 \), \( g(x) = -4x + 3 \), and \( h(x) = x^2 - 2x + 1 \), find each value.

53. \([ (f + g) \cdot h ](1)\)

**ANSWER:**

0

55. \([ f \circ (g \circ h)](2)\)

**ANSWER:**

1

57. \([ h \circ (f \circ g)](5)\)

**ANSWER:**

256

59. **OPEN ENDED** Write two functions \( f(x) \) and \( g(x) \) such that \((f \circ g)(4) = 0\).

**ANSWER:**

Sample answer: \( f(x) = x - 9 \), \( g(x) = x + 5 \)

61. **CHALLENGE** Given \( f(x) = \sqrt{x^3} \) and \( g(x) = \sqrt{x^6} \), determine the domain for each of the following.

a. \( g(x) \cdot g(x) \)

b. \( f(x) \cdot f(x) \)

**ANSWER:**

a. \( D = \{ \text{all real numbers} \} \)

b. \( D = \{ x \mid x \geq 0 \} \)

63. **WRITING IN MATH** In the real world, why would you ever perform a composition of functions?

**ANSWER:**

Sample answer: Many situations in the real world involve complex calculations in which multiple functions are used. In order to solve some problems, a composition of those functions may need to be used. For example, the product of a manufacturing plant may have to go through several processes in a particular order, in which each process is described by a function. By finding the composition, only one calculation must be made to find the solution to the problem.
6-1 Operations on Functions

65. If \( g(x) = x^2 + 9x + 21 \) and \( h(x) = 2(x + 5)^2 \), which is an equivalent form of \( h(x) - g(x) \)?

\[ F \ k(x) = -x^2 - 11x - 29 \]
\[ G \ k(x) = x^2 + 11x + 29 \]
\[ H \ k(x) = x + 4 \]
\[ J \ k(x) = x^2 + 7x + 11 \]

**ANSWER:**

**G**

67. SAT/ACT What is the value of \( f \left( g(6) \right) \) if \( f(x) = 2x + 4 \) and \( g(x) = x^2 + 5 \)?

\[ A \ 38 \]
\[ B \ 43 \]
\[ C \ 57 \]
\[ D \ 86 \]
\[ E \ 261 \]

**ANSWER:**

**D**

Find all rational zeros of each function.

69. \( f(x) = x^3 - 3x^2 - 10x + 24 \)

**ANSWER:**

\(-3, 2, 4\)

71. \( f(x) = 2x^3 - 5x^2 - 28x + 15 \)

**ANSWER:**

\(-3, 5, \frac{1}{2}\)

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

73. \( f(x) = -4x^4 - x^2 - x + 1 \)

**ANSWER:**

1; 1; 2

75. \( f(x) = 2x^4 - 3x^3 - 2x^2 + 3 \)

**ANSWER:**

2 or 0; 2 or 0; 4, 2, or 0

Solve each system of equations.

77. \( 3x + 2y + 3z = 16 \)
\( 2x - y + z = 3 \)

**ANSWER:**

\((1, 2, 3)\)

79. \( 2x + 4y - z = -3 \)
\( y + z = 4 \)
\( 3y = -3 \)

**ANSWER:**

\((3, -1, 5)\)
6-1 Operations on Functions

Solve each equation or formula for the specified variable.

81. \(5x - 7y = 12\), for \(x\)

\[\text{ANSWER:} \quad x = \frac{12 + 7y}{5}\]

83. \(4x + 8yz = 15\), for \(x\)

\[\text{ANSWER:} \quad x = \frac{15 - 8yz}{4}\]

85. \(A = k^2 + b\), for \(k\)

\[\text{ANSWER:} \quad k = \pm \sqrt{A - b}\]