Write each equation in exponential form.

1. $\log_8 512 = 3$

SOLUTION:

$$\log_8 512 = 3$$
$$8^3 = 512$$

ANSWER:

$8^3 = 512$

2. $\log_5 625 = 4$

SOLUTION:

$$\log_5 625 = 4$$
$$5^4 = 625$$

ANSWER:

$5^4 = 625$

Write each equation in logarithmic form.

3. $11^3 = 1331$

SOLUTION:

$$11^3 = 1331$$
$$\log_{11} 1331 = 3$$

ANSWER:

$\log_{11} 1331 = 3$

4. $16^{\frac{3}{4}} = 8$

SOLUTION:

$$16^{\frac{3}{4}} = 8$$
$$\log_{16} 8 = \frac{3}{4}$$

ANSWER:

$\log_{16} 8 = \frac{3}{4}$

Evaluate each expression.

5. $\log_{13} 169$

SOLUTION:

$$\log_{13} 169 = \log_{13}\left(13^2\right)$$
$$= 2$$

ANSWER:

2

6. $\log_2 \frac{1}{128}$

SOLUTION:

$$\log_2 \frac{1}{128} = \log_2 \frac{1}{2^7}$$
$$= \log_2 2^{-7}$$
$$= -7$$

ANSWER:

–7

7. $\log_6 1$

SOLUTION:

$\log_6 1 = 0$

ANSWER:

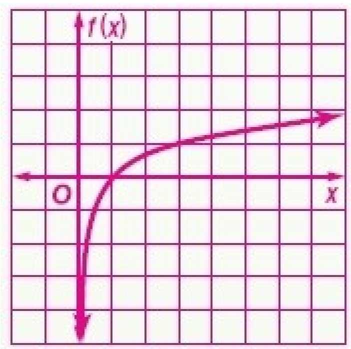
0

Graph each function. State the domain and range.

8. $f(x) = \log_3 x$

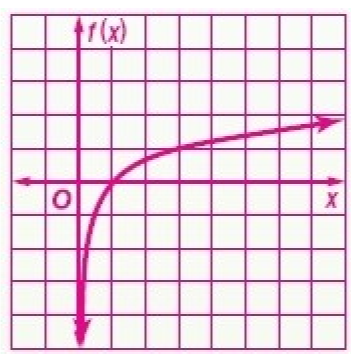
SOLUTION:

Plot the points $\left(\frac{1}{3}, -1\right), (1, 0), (3, 1)$ and sketch the graph.



The domain consists of all positive real numbers, and the domain consists of all real numbers.

ANSWER:

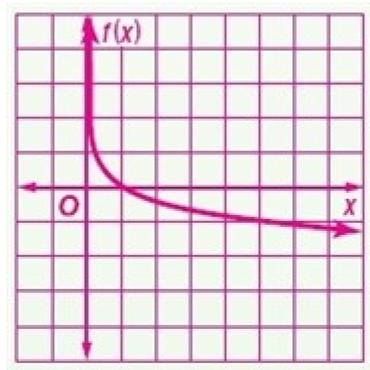


D = {x | $x > 0$}; R = {all real numbers)

9. $f(x) = \log_{\frac{1}{6}} x$

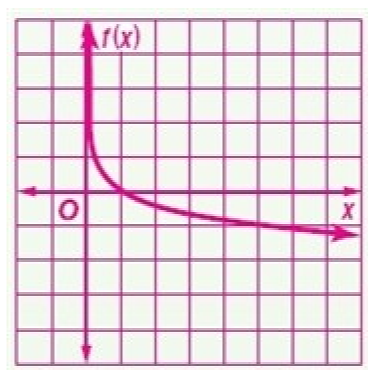
SOLUTION:

Plot the points $(6, -1), (1, 0), \left(\frac{1}{6}, 1\right)$ and sketch the graph.



The domain consists of all positive real numbers, and the domain consists of all real numbers.

ANSWER:



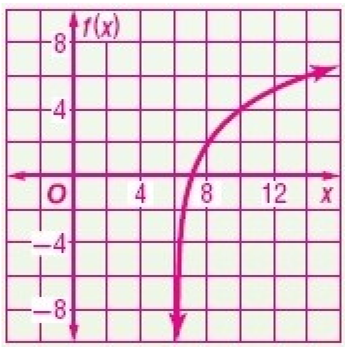
D = {x | $x > 0$); R = {all real numbers}

10. $f(x) = 4\log_4(x - 6)$

SOLUTION:

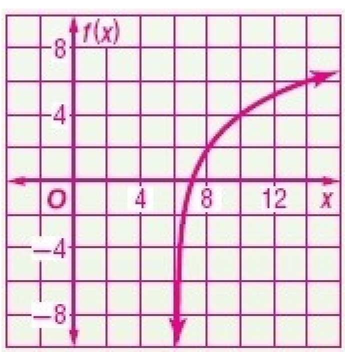
The function represents a transformation of the graph of $f(x) = \log_4 x$.

a = 4: The graph expands vertically.
h = 6: The graph is translated 6 units to the right.
k = 0: There is no vertical shift.



The domain consists of all positive real numbers greater than 6, and the domain consists of all real numbers.

ANSWER:



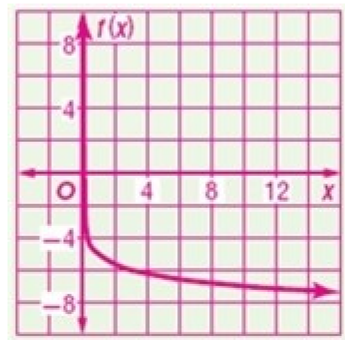
D = {x | x >6}; R = {all real numbers}

11. $f(x) = 2\log_{\frac{1}{10}} x - 5$

SOLUTION:

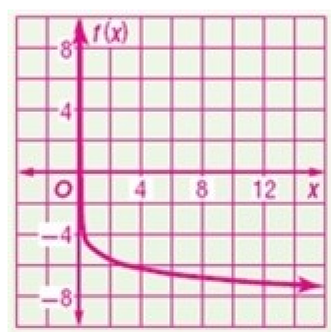
The function represents a transformation of the graph of $f(x) = \log_{\frac{1}{10}} x$.

a = 2: The graph expands vertically.
h = 0: There is no horizontal shift.
k = –5: The graph is translated 5 units down.



The domain consists of all positive real numbers, and the domain consists of all real numbers.

ANSWER:



D = {x | x > 0}; R = {all real numbers}

12. **SCIENCE** Use the information at the beginning of the lesson. The Palermo scale value of any object can be found using the equation $PS = \log_{10} R$, where R is the relative risk posed by the object. Write an equation in exponential form for the inverse of the function.

SOLUTION:

Rewrite the equation in exponential form.

$10^{PS} = R$

Interchange the variables.

$PS = 10^R$

ANSWER:

$PS = 10^R$

Write each equation in exponential form.

13. $\log_2 16 = 4$

SOLUTION:

$$\log_2 16 = 4$$
$$2^4 = 16$$

ANSWER:

$2^4 = 16$

14. $\log_7 343 = 3$

SOLUTION:

$$\log_7 343 = 3$$
$$7^3 = 343$$

ANSWER:

$7^3 = 343$

15. $\log_9 \frac{1}{81} = -2$

SOLUTION:

$$\log_9 \frac{1}{81} = -2$$
$$9^{-2} = \frac{1}{81}$$

ANSWER:

$9^{-2} = \frac{1}{81}$

16. $\log_3 \frac{1}{27} = -3$

SOLUTION:

$$\log_3 \frac{1}{27} = -3$$
$$3^{-3} = \frac{1}{27}$$

ANSWER:

$3^{-3} = \frac{1}{27}$

17. $\log_{12} 144 = 2$

SOLUTION:

$$\log_{12} 144 = 2$$
$$12^2 = 144$$

ANSWER:

$12^2 = 144$

18. $\log_9 1 = 0$

SOLUTION:

$$\log_9 1 = 0$$
$$9^0 = 1$$

ANSWER:

$9^0 = 1$

Write each equation in logarithmic form.

19. $9^{-1} = \frac{1}{9}$

SOLUTION:

$$\log_9 \frac{1}{9} = -1$$

ANSWER:

$\log_9 \frac{1}{9} = -1$

20. $6^{-3} = \frac{1}{216}$

SOLUTION:

$\log_6 \frac{1}{216} = -3$

ANSWER:

$\log_6 \frac{1}{216} = -3$

21. $2^8 = 256$

SOLUTION:

$\log_2 256 = 8$

ANSWER:

$\log_2 256 = 8$

22. $4^6 = 4096$

SOLUTION:

$\log_4 4096 = 6$

ANSWER:

$\log_4 4096 = 6$

23. $27^{\frac{2}{3}} = 9$

SOLUTION:

$\log_{27} 9 = \frac{2}{3}$

ANSWER:

$\log_{27} 9 = \frac{2}{3}$

24. $25^{\frac{3}{2}} = 125$

SOLUTION:

$\log_{25} 125 = \frac{3}{2}$

ANSWER:

$\log_{25} 125 = \frac{3}{2}$

Evaluate each expression.

25. $\log_3 \frac{1}{9}$

SOLUTION:

$$\begin{aligned}\log_3 \frac{1}{9} &= \log_3 \frac{1}{3^2} \\ &= \log_3 3^{-2} \\ &= -2\end{aligned}$$

ANSWER:

-2

26. $\log_4 \frac{1}{64}$

SOLUTION:

$$\begin{aligned}\log_4 \frac{1}{64} &= \log_4 \frac{1}{4^3} \\ &= \log_4 4^{-3} \\ &= -3\end{aligned}$$

ANSWER:

-3

27. $\log_8 512$

SOLUTION:

$$\begin{aligned}\log_8 512 &= \log_8 8^3 \\ &= 3\end{aligned}$$

ANSWER:

3

28. $\log_6 216$

SOLUTION:

$$\begin{aligned}\log_6 216 &= \log_6 6^3 \\ &= 3\end{aligned}$$

ANSWER:

3

29. $\log_{27} 3$

SOLUTION:
Let y be the unknown value.

$$\begin{aligned} \log_{27} 3 &= y \\ 27^y &= 3 \\ 3^{3y} &= 3^1 \\ 3y &= 1 \\ y &= \frac{1}{3} \end{aligned}$$

ANSWER:
$\frac{1}{3}$

30. $\log_{32} 2$

SOLUTION:
Let y be the unknown value.

$$\begin{aligned} \log_{32} 2 &= y \\ 32^y &= 2 \\ 2^{5y} &= 2^1 \\ 5y &= 1 \\ y &= \frac{1}{5} \end{aligned}$$

ANSWER:
$\frac{1}{5}$

31. $\log_9 3$

SOLUTION:
Let y be the unknown value.

$$\begin{aligned} \log_9 3 &= y \\ 9^y &= 3 \\ 3^{2y} &= 3^1 \\ 2y &= 1 \\ y &= \frac{1}{2} \end{aligned}$$

ANSWER:
$\frac{1}{2}$

32. $\log_{121} 11$

SOLUTION:
Let y be the unknown value.

$$\begin{aligned} \log_{121} 11 &= y \\ 121^y &= 11 \\ 11^{2y} &= 11^1 \\ 2y &= 1 \\ y &= \frac{1}{2} \end{aligned}$$

ANSWER:
$\frac{1}{2}$

33. $\log_{\frac{1}{5}} 3125$

SOLUTION:
Let y be the unknown value.

$$\begin{aligned} \log_{\frac{1}{5}} 3125 &= y \\ \left(\frac{1}{5}\right)^y &= 3125 \\ 5^{-y} &= 5^5 \\ -y &= 5 \\ y &= -5 \end{aligned}$$

ANSWER:
–5

34. $\log_{\frac{1}{8}} 512$

SOLUTION:
Let y be the unknown value.

$$\begin{aligned} \log_{\frac{1}{8}} 512 &= y \\ \left(\frac{1}{8}\right)^y &= 512 \\ 8^{-y} &= 8^3 \\ -y &= 3 \\ y &= -3 \end{aligned}$$

ANSWER:
–3

35. $\log_{\frac{1}{3}} \frac{1}{81}$

SOLUTION:

$$\log_{\frac{1}{3}} \frac{1}{81} = \log_{\frac{1}{3}} \frac{1}{3^4}$$
$$= \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^4$$
$$= 4$$

ANSWER:
4

36. $\log_{\frac{1}{6}} \frac{1}{216}$

SOLUTION:

$$\log_{\frac{1}{6}} \frac{1}{216} = \log_{\frac{1}{6}} \frac{1}{6^3}$$
$$= \log_{\frac{1}{6}} \left(\frac{1}{6}\right)^3$$
$$= 3$$

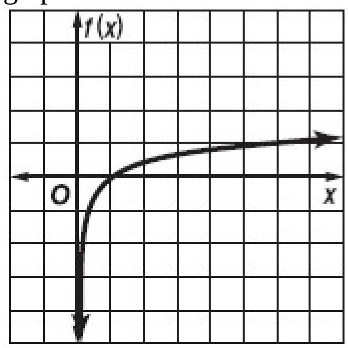
ANSWER:
3

CCSS PRECISION Graph each function.

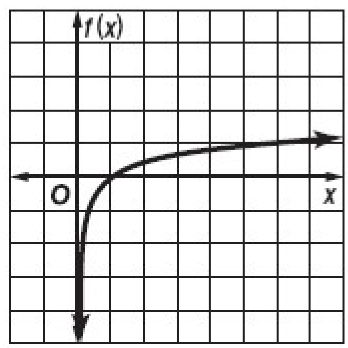
37. $f(x) = \log_6 x$

SOLUTION:

Plot the points $\left(\frac{1}{6}, -1\right), (1, 0), (6, 1)$ and sketch the graph.



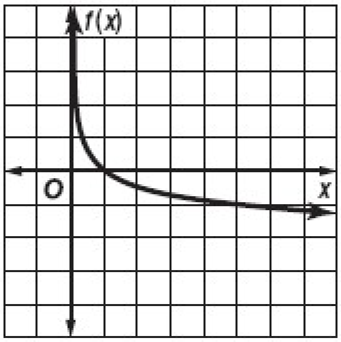
ANSWER:



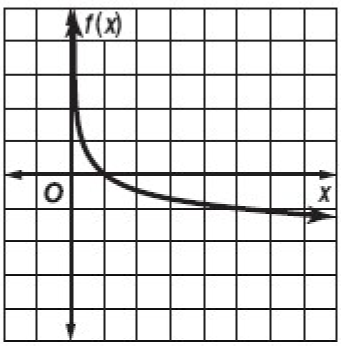
38. $f(x) = \log_{\frac{1}{5}} x$

SOLUTION:

Plot the points $(5,-1),(1,0),\left(\frac{1}{5},1\right)$ and sketch the graph.



ANSWER:

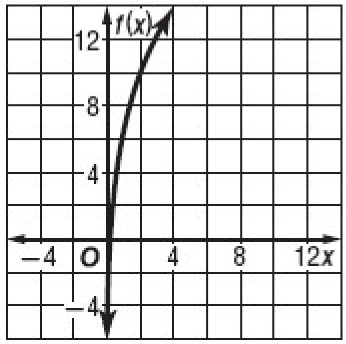


39. $f(x) = 4\log_2 x + 6$

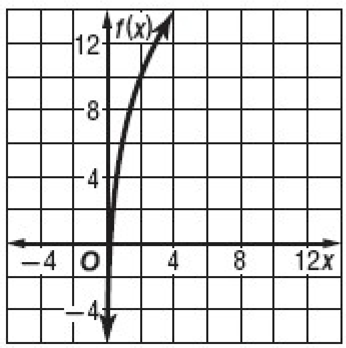
SOLUTION:

The function represents a transformation of the graph of $f(x) = \log_2 x$.

$a = 4$: The graph expands vertically.
$h = 0$: There is no horizontal shift.
$k = 6$: The graph is translated 6 units up.



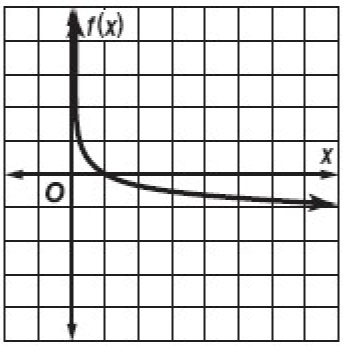
ANSWER:



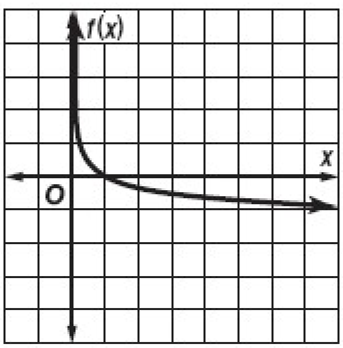
40. $f(x) = \log_{\frac{1}{9}} x$

SOLUTION:

Plot the points $(9, -1), (1, 0), \left(\frac{1}{9}, 1\right)$ and sketch the graph.



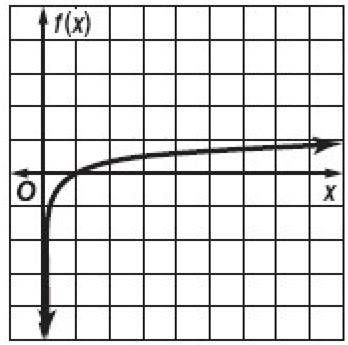
ANSWER:



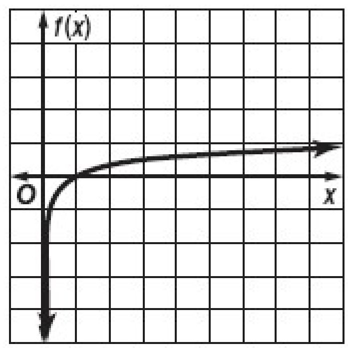
41. $f(x) = \log_{10} x$

SOLUTION:

Plot the points $\left(\frac{1}{10}, -1\right), (1, 0), (10, 1)$ and sketch the graph.



ANSWER:

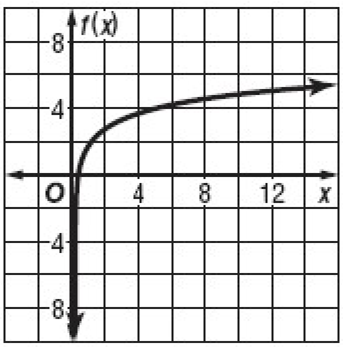


42. $f(x) = -3\log_{\frac{1}{12}} x + 2$

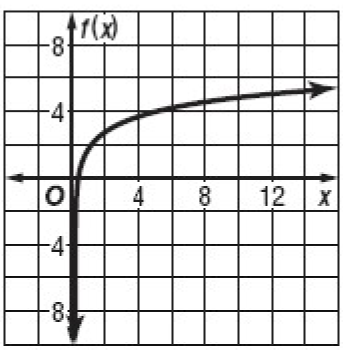
SOLUTION:

The function represents a transformation of the graph of $f(x) = \log_{\frac{1}{12}} x$.

a = –3: The graph is reflected across the x–axis.
h = 0: There is no horizontal shift.
k = 2: The graph is translated 2 units up.



ANSWER:

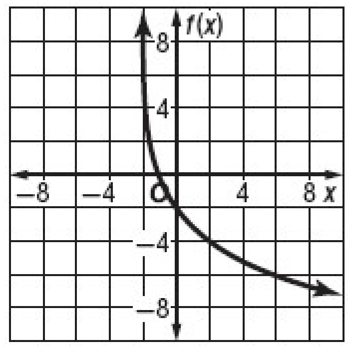


43. $f(x) = 6\log_{\frac{1}{8}}(x+2)$

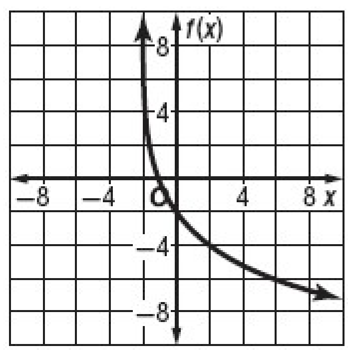
SOLUTION:

The function represents a transformation of the graph of $f(x) = \log_{\frac{1}{8}} x$.

a = 6: The graph expands vertically.
h = –2: The graph is translated 2 units to the left.
k = 0: There is no vertical shift.



ANSWER:

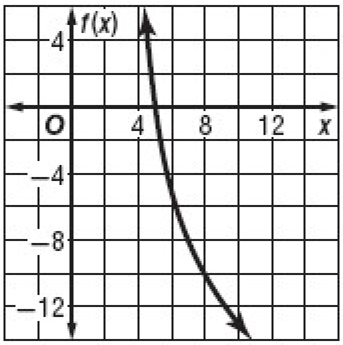


44. $f(x) = -8 \log_3 (x - 4)$

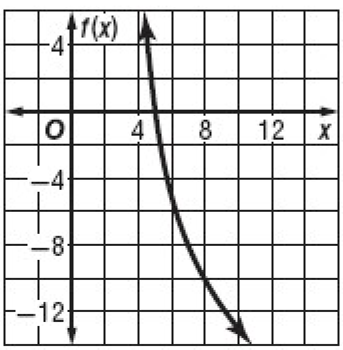
SOLUTION:

The function represents a transformation of the graph of $f(x) = \log_3 x$.

$a = -8$: The graph is reflected across the x–axis.
$h = 4$: The graph is translated 4 units to the right.
$k = 0$: There is no vertical shift.



ANSWER:

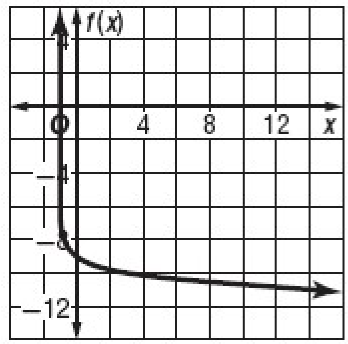


45. $f(x) = \log_{\frac{1}{4}} (x + 1) - 9$

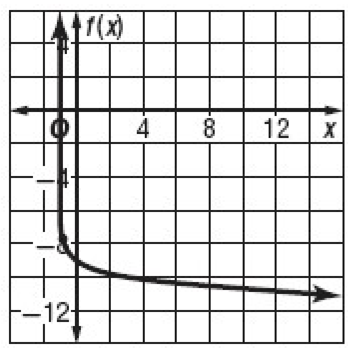
SOLUTION:

The function represents a transformation of the graph of $f(x) = \log_{\frac{1}{4}} x$.

$h = -1$: The graph is translated 1 unit to the left.
$k = -9$: The graph is translated 9 units down.



ANSWER:

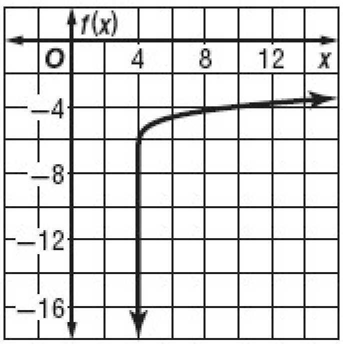


46. $f(x) = \log_5 (x - 4) - 5$

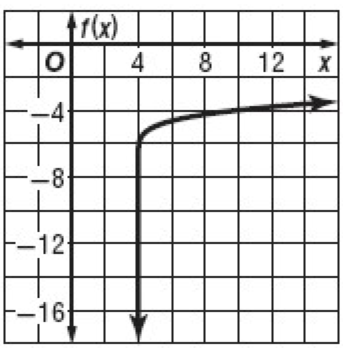
SOLUTION:

The function represents a transformation of the graph of $f(x) = \log_5 x$.

$h = 4$: The graph is translated 4 units to the right.
$k = -5$: The graph is translated 5 units down.



ANSWER:

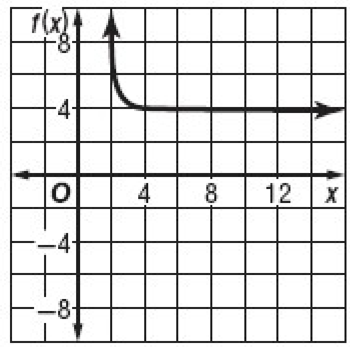


47. $f(x) = -\frac{1}{6}\log_8 (x - 3) + 4$

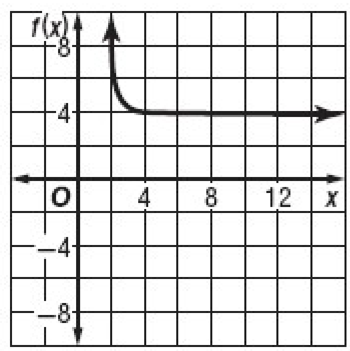
SOLUTION:

The function represents a transformation of the graph of $f(x) = \log_8 x$.

$a = -\frac{1}{6}$: The graph is reflected across the x–axis.
$h = 3$: The graph is translated 3 units to the right.
$k = 4$: The graph is translated 4 units up.



ANSWER:



48. $f(x) = -\frac{1}{3}\log_{\frac{1}{6}}(x+2) - 5$

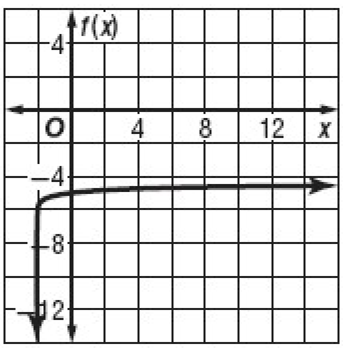
SOLUTION:

The function represents a transformation of the graph of $f(x) = \log_{\frac{1}{6}} x$.

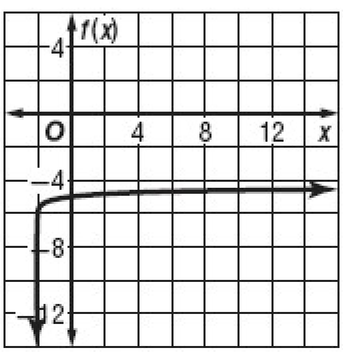
$a = -\frac{1}{3}$: The graph is reflected across the *x*–axis.

$h = -2$: The graph is translated 2 units to the left.

$k = -5$: The graph is translated 5 units down.



ANSWER:



49. **PHOTOGRAPHY** The formula $n = \log_2 \frac{1}{p}$ represents the change in the f-stop setting *n* to use in less light where p is the fraction of sunlight.

a. Benito's camera is set up to take pictures in direct sunlight, but it is a cloudy day. If the amount of sunlight on a cloudy day is $\frac{1}{4}$ as bright as direct sunlight, how many f-stop settings should he move to accommodate less light?

b. Graph the function.

c. Use the graph in part b to predict what fraction of daylight Benito is accommodating if he moves down 3 f-stop settings. Is he allowing more or less light into the camera?

SOLUTION:

a.

Substitute $\frac{1}{4}$ for p in the formula and simplify.

$$n = \log_2 \frac{1}{p}$$

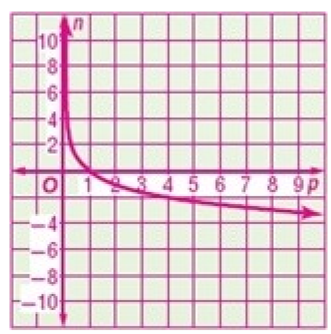
$$\begin{aligned} n &= \log_2 \frac{1}{\frac{1}{4}} \\ &= \log_2 4 \\ &= \log_2 2^2 \\ &= 2 \end{aligned}$$

b.

$$\begin{aligned} n &= \log_2 \frac{1}{p} \\ &= \log_2 1 - \log_2 p \\ &= 0 - \log_2 p \\ &= -\log_2 p \end{aligned}$$

The function represents a transformation of the graph of $f(x) = \log_2 x$.

$a = -1$: The graph is reflected across the *x*–axis.



c.

Substitute 3 for *n* in the formula and solve for p.

$$3 = \log_2 \frac{1}{p}$$

$$2^3 = \frac{1}{p}$$

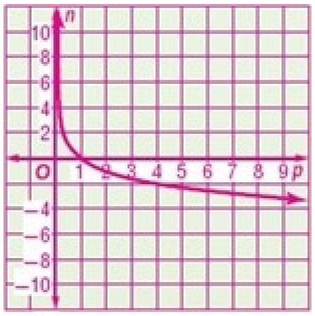
$$p = \frac{1}{8}$$

As $\frac{1}{4} > \frac{1}{8}$, he is allowing less light into the camera.

ANSWER:

a. 2

b.



c. $\frac{1}{8}$; less light

50. **EDUCATION** To measure a student's retention of knowledge, the student is tested after a given amount of time. A student's score on an Algebra 2 test t months after the school year is over can be approximated by $y(t) = 85 - 6\log_2(t + 1)$, where $y(t)$ is the student's score as a percent.

a. What was the student's score at the time the school year ended ($t = 0$)?

b. What was the student's score after 3 months?

c. What was the student's score after 15 months?

SOLUTION:

a.

Substitute 0 for t in the function and simplify.

$$\begin{aligned} y(t) &= 85 - 6\log_2(0+1) \\ &= 85 - 6\log_2 1 \\ &= 85 - 0 \\ &= 85 \end{aligned}$$

b.

Substitute 2 for t in the function and simplify.

$$\begin{aligned} y(t) &= 85 - 6\log_2(3+1) \\ &= 85 - 6\log_2 4 \\ &= 73 \end{aligned}$$

c.

Substitute 15 for t in the function and simplify.

$$\begin{aligned} y(t) &= 85 - 6\log_2(15+1) \\ &= 85 - 6\log_2 16 \\ &= 61 \end{aligned}$$

ANSWER:

a. 85

b. 73

c. 61

Graph each function.

51. $f(x) = 4\log_2(2x - 4) + 6$

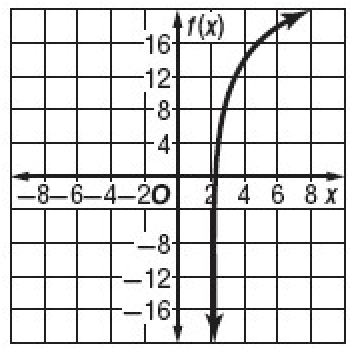
SOLUTION:

The function represents a transformation of the graph of $f(x) = \log_2 2x$.

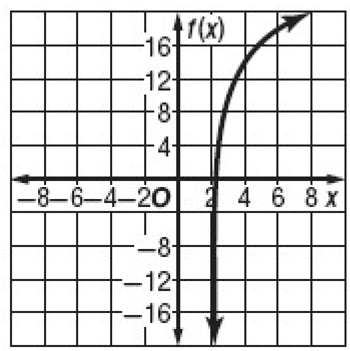
$a = 4$: The graph expands vertically.

$h = 4$: The graph is translated 4 units to the right.

$k = 6$: The graph is translated 6 units up.



ANSWER:

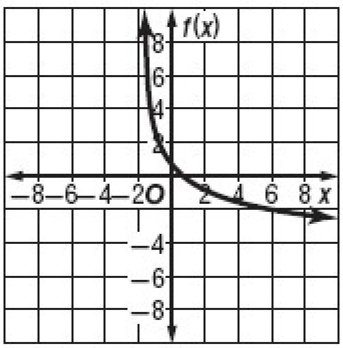


52. $f(x) = -3 \log_{12}(4x + 3) + 2$

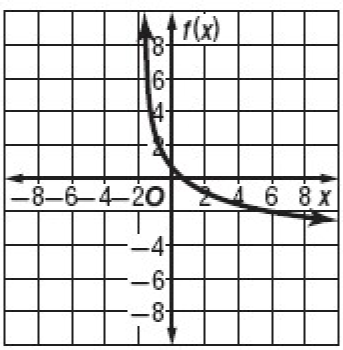
SOLUTION:

The function represents a transformation of the graph of $f(x) = \log_{12} 4x$.

a = –3: The graph is reflected across the x-axis.
h = –3: The graph is translated 3 units to the left.
k = 2: The graph is translated 2 units up.



ANSWER:

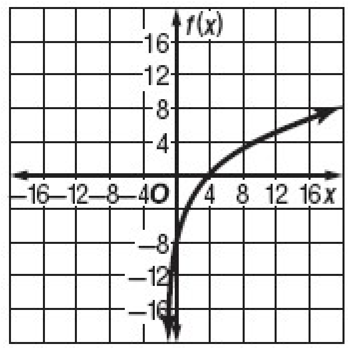


53. $f(x) = 15 \log_{14}(x + 1) - 9$

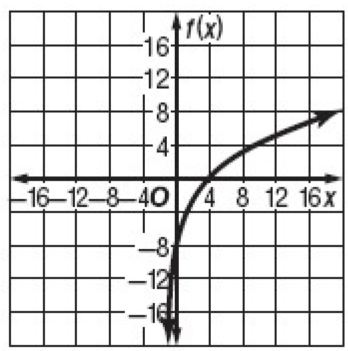
SOLUTION:

The function represents a transformation of the graph of $f(x) = \log_{14} x$.

a = 15: The graph expands vertically.
h = –1: The graph is translated 1 unit to the left.
k = –9: The graph is translated 9 units down.



ANSWER:

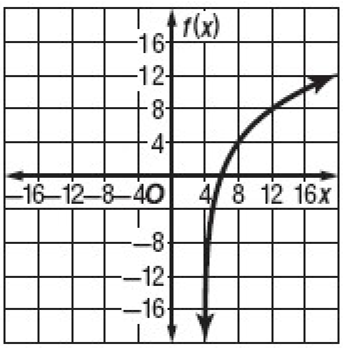


54. $f(x) = 10 \log_5 (x - 4) - 5$

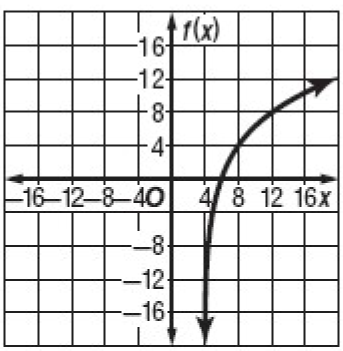
SOLUTION:

The function represents a transformation of the graph of $f(x) = \log_5 x$.

$a = 10$: The graph expands vertically.
$h = 4$: The graph is translated 4 units to the right.
$k = -5$: The graph is translated 5 units down.



ANSWER:

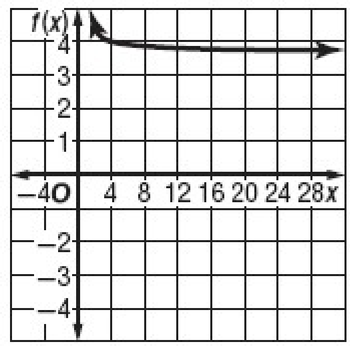


55. $f(x) = -\frac{1}{6}\log_8 (x-3) + 4$

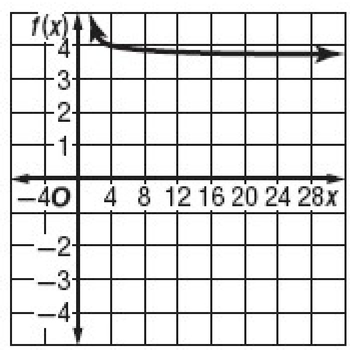
SOLUTION:

The function represents a transformation of the graph of $f(x) = \log_8 x$.

$a = -\frac{1}{6}$: The graph is reflected across the *x*-axis.
$h = 4$: The graph is translated 4 units to the right.
$k = -5$: The graph is translated 5 units down.



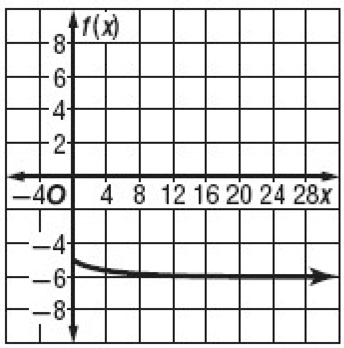
ANSWER:



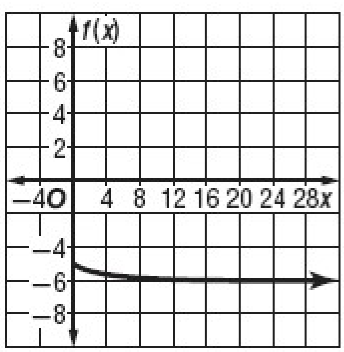
56. $f(x) = -\frac{1}{3}\log_6(6x+2) - 5$

SOLUTION:
The function represents a transformation of the graph of $f(x) = \log_6 6x$.

$a = -\frac{1}{3}$: The graph is reflected across the x-axis.
$h = -2$: The graph is translated 2 units to the left.
$k = -5$: The graph is translated 5 units down.



ANSWER:



57. **CCSS MODELING** In general, the more money a company spends on advertising, the higher the sales. The amount of money in sales for a company, in thousands, can be modeled by the equation $S(a) = 10 + 20\log_4(a + 1)$, where a is the amount of money spent on advertising in thousands, when $a \geq 0$.
a. The value of $S(0) \approx 10$, which means that if \$10 is spent on advertising, \$10,000 is returned in sales. Find the values of $S(3)$, $S(15)$, and $S(63)$.
b. Interpret the meaning of each function value in the context of the problem.
c. Graph the function.
d. Use the graph in part c and your answers from part a to explain why the money spent in advertising becomes less "efficient" as it is used in larger amounts.

SOLUTION:
a.
Substitute 3 for a in the equation and simplify.

$$s(3) = 10 + 20\log_4(3+1) \\ = 30$$

Substitute 15 for a in the equation and simplify.

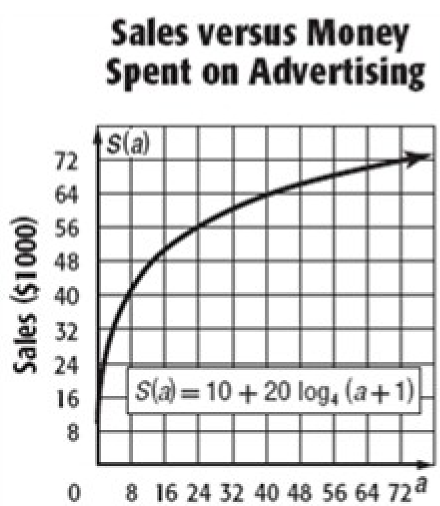
$$s(15) = 10 + 20\log_4(15+1) \\ = 50$$

Substitute 63 for a in the equation and simplify.

$$s(63) = 10 + 20\log_4(63+1) \\ = 70$$

b. If \$3000 is spent on advertising, \$30,000 is returned in sales. If \$15,000 is spent on advertising, \$50,000 is returned in sales. If \$63,000 is spent on advertising, \$70,000 is returned in sales.
c.
The function represents a transformation of the graph of $f(x) = \log_4 x$.

$a = 20$: The graph is expanded vertically.
$h = -1$: The graph is translated 1 unit to the left.
$k = 10$: The graph is translated 10 units up.



d.

Because eventually the graph plateaus and no matter how much money you spend you are still returning about the same in sales.

ANSWER:

a. S(3) =30, S(15) = 50, S(63) = 70
b. If $3000 is spent on advertising, $30,000 is returned in sales. If $15,000 is spent on advertising, $50,000 is returned in sales. If $63,000 is spent on advertising, $70,000 is returned in sales.
c.

d. Because eventually the graph plateaus and no matter how much money you spend you are still returning about the same in sales.

58. **BIOLOGY** The generation time for bacteria is the time that it takes for the population to double. The generation time G for a specific type of bacteria can be found using experimental data and the formula $G = \frac{t}{3.3 \log_b f}$, where t is the time period, b is the number of bacteria at the beginning of the experiment, and f is the number of bacteria at the end of the experiment.

a. The generation time for mycobacterium tuberculosis is 16 hours. How long will it take four of these bacteria to multiply into 1024 bacteria?
b. An experiment involving rats that had been exposed to salmonella showed that the generation time for the salmonella was 5 hours. After how long would 20 of these bacteria multiply into 8000?
c. E. coli are fast growing bacteria. If 6 e. coli can grow to 1296 in 4.4 hours, what is the generation time of e. coli?

SOLUTION:

a. Substitute G = 16, b = 4, and f = 1024 into the bacterial growth formula.

$$G = \frac{t}{3.3 \log_b f}$$
$$16 = \frac{t}{3.3 \log_4 1024}$$
$$52.8 \log_4 1024 = t$$
$$52.8 \cdot 5 = t$$
$$264 = t$$

Therefore, t = 264 hours or 11 days.

b. Substitute G = 5, b = 20, and f = 8000 into the bacterial growth formula.

$$G = \frac{t}{3.3 \log_b f}$$
$$5 = \frac{t}{3.3 \log_{20} 8000}$$
$$16.5 \log_{20} 8000 = t$$
$$16.5 \cdot 3 = t$$
$$49.5 = t$$

Therefore, t = 49.5 hours or about 2 days 1.5 hours.

c. Substitute t = 4.4, b = 6, and f = 1296 into the bacterial growth formula.

$$G = \frac{t}{3.3 \log_b f}$$
$$= \frac{4.4}{3.3 \log_6 1296}$$
$$= \frac{4.4}{3.3 \cdot 4}$$
$$\approx 0.333$$

Therefore, $G = \frac{1}{3}$ hour or 20 minutes.

ANSWER:

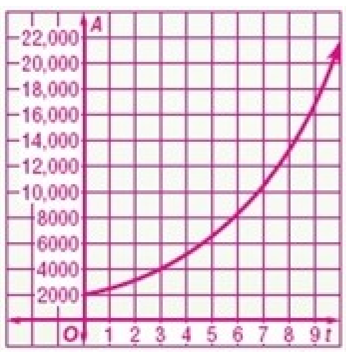
a. 264 h or 11 days
b. 49.5 h or about 2 days 1.5 h
c. $\frac{1}{3}$ h or 20 min

59. **FINANCIAL LITERACY** Jacy has spent $2000 on a credit card. The credit card company charges 24% interest, compounded monthly. The credit card company uses $\log_{\left(1+\frac{0.24}{12}\right)} \frac{A}{2000} = 12t$ to determine how much time it will be until Jacy's debt reaches a certain amount, if A is the amount of debt after a period of time, and t is time in years.

a. Graph the function for Jacy's debt.
b. Approximately how long will it take Jacy's debt to double?
c. Approximately how long will it be until Jacy's debt triples?

SOLUTION:

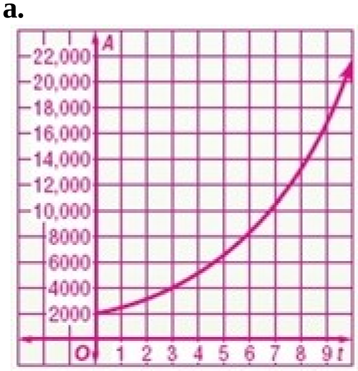
a.
Graph of the function for Jacy's debt:



b. It will take about 3 years for Jacy's debt to double.
c. It will take about 4 years for Jacy's debt to triple.

ANSWER:

a.



b. ≈ 3 years
c. ≈ 4.5 years

60. **WRITING IN MATH** What should you consider when using exponential and logarithmic models to make decisions?

SOLUTION:
Sample answer: Exponential and logarithmic models can grow without bound, which is usually not the case of the situation that is being modeled. For instance, a population cannot grow without bound due to space and food constraints. Therefore, when using a model to make decisions, the situation that is being modeled should be carefully considered.

ANSWER:
Sample answer: Exponential and logarithmic models can grow without bound, which is usually not the case of the situation that is being modeled. For instance, a population cannot grow without bound due to space and food constraints. Therefore, when using a model to make decisions, the situation that is being modeled should be carefully considered.

61. **CCSS ARGUMENTS** Consider $y = \log_b x$ in which b, x, and y are real numbers. Zero can be in the domain *sometimes*, *always* or *never*. Justify your answer.

SOLUTION:
Never; if zero were in the domain, the equation would be $y = \log_b 0$. Then $b^y = 0$. However, for any real number b, there is no real power that would let $b^y = 0$

ANSWER:
Never; if zero were in the domain, the equation would be $y = \log_b 0$. Then $b^y = 0$. However, for any real number b, there is no real power that would let $b^y = 0$

62. **ERROR ANALYSIS** Betsy says that the graphs of all logarithmic functions cross the y-axis at (0, 1) because any number to the zero power equals 1. Tyrone disagrees. Is either of them correct? Explain your reasoning.

SOLUTION:

Tyrone; sample answer: The graphs of logarithmic functions pass through (1, 0) not (0, 1).

ANSWER:

Tyrone; sample answer: The graphs of logarithmic functions pass through (1, 0) not (0, 1).

63. **REASONING** Without using a calculator, compare $\log_7 51$, $\log_8 61$, and $\log_9 71$. Which of these is the greatest? Explain your reasoning.

SOLUTION:

$\log_7 51$; Sample answer: $\log_7 51$ equals a little more than 2. $\log_8 61$ equals a little less than 2. $\log_9 71$ equals a little less than 2. Therefore, $\log_7 51$ is the greatest.

ANSWER:

$\log_7 51$; sample answer: $\log_7 51$ equals a little more than 2. $\log_8 61$ equals a little less than 2. $\log_9 71$ equals a little less than 2. Therefore, $\log_7 51$ is the greatest.

64. **OPEN ENDED** Write a logarithmic expression of the form $y = \log_b x$ for each of the following conditions.

a. y is equal to 25.
b. y is negative.
c. y is between 0 and 1.
d. x is 1.
e. x is 0.

SOLUTION:

Sample answers:

a. $\log_2 33{,}554{,}432 = 25$;

b. $\log_4 \frac{1}{64} = -3$;

c. $\log_2 \sqrt{2} = \frac{1}{2}$

d. $\log_7 1 = 0$;

e. There is no possible solution; this is the empty set.

ANSWER:

Sample answers:

a. $\log_2 33{,}554{,}432 = 25$;

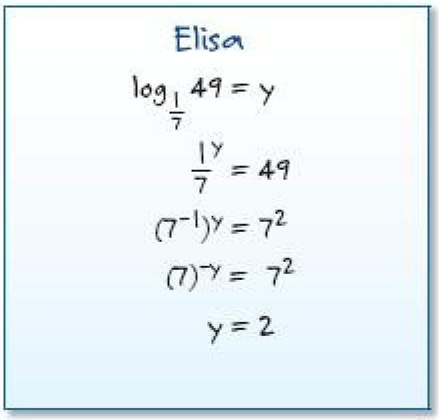
b. $\log_4 \frac{1}{64} = -3$;

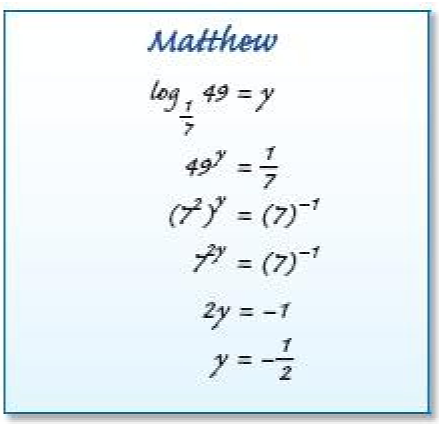
c. $\log_2 \sqrt{2} = \frac{1}{2}$

d. $\log_7 1 = 0$;

e. There is no possible solution; this is the empty set.

65. **FIND THE ERROR** Elisa and Matthew are evaluating $\log_{\frac{1}{7}} 49$ Is either of them correct? Explain your reasoning.





SOLUTION:

No; Elisa was closer. She should have $–y = 2$ or $y = –2$ instead of $y = 2$. Matthew used the definition of logarithms incorrectly.

ANSWER:

No; Elisa was closer. She should have $–y = 2$ or $y = –2$ instead of $y = 2$. Matthew used the definition of logarithms incorrectly.

66. **WRITING IN MATH** A transformation of $\log_{10} x$ is $g(x) = a\log_{10}(x - h) + k$. Explain the process of graphing this transformation.

SOLUTION:

Sample answer: In $g(x) = a\log_{10}(x - h) + k$, the value of k is a vertical translation and the graph will shift up k units if k is positive and down $|k|$ units if k is negative. The value of h is a horizontal translation and the graph will shift h units to the right if h is positive and $|h|$ units to the left if h is negative. If $a < 0$, the graph will be reflected across the x-axis. if $|a| > 1$, the graph will be expanded vertically and if $0 < |a| < 1$, then the graph will be compressed vertically.

ANSWER:

Sample answer: In $g(x) = a\log_{10}(x - h) + k$, the value of k is a vertical translation and the graph will shift up k units if k is positive and down $|k|$ units if k is negative. The value of h is a horizontal translation and the graph will shift h units to the right if h is positive and $|h|$ units to the left if h is negative. If $a < 0$, the graph will be reflected across the x-axis. if $|a| > 1$, the graph will be expanded vertically and if $0 < |a| < 1$, then the graph will be compressed vertically.

67. A rectangle is twice as long as it is wide. If the width of the rectangle is 3 inches, what is the area of the rectangle in square inches?

A 9
B 12
C 15
D 18

SOLUTION:

Length of the rectangle = 2 * 3 = 6 inches.
Area of the rectangle = 6 * 3 = 18 square inches.
D is the correct option.

ANSWER:

D

68. **SAT/ACT** Ichiro has some pizza. He sold 40% more slices than he ate. If he sold 70 slices of pizza, how many did he eat?
F 25
G 50
H 75
J 98
K 100

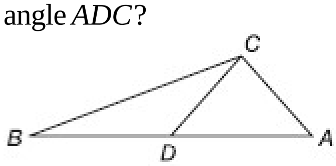
SOLUTION:
Let x be the number of pizza slices Ichiro ate.
The equation that represents the situation is:

$$x + 0.4x = 70$$
$$1.4x = 70$$
$$x = 50$$

G is the correct answer.

ANSWER:
G

69. **SHORT RESPONSE** In the figure $AB = BC$, $CD = BD$, and angle $CAD = 70°$. What is the measure of angle ADC?



SOLUTION:
ΔABC and ΔDBC are isosceles triangles.
In ΔABC, $\angle BCA = 70°$ and $\angle ABC = 40°$.
In ΔDBC, $\angle DBC = 40°$ and $\angle BCD = 40°$.
So, $\angle ACD = 30°$.
Thus, $\angle ADC = 80°$.

ANSWER:
80

70. If $6x - 3y = 30$ and $4x = 2 - y$ then find $x + y$.
A −4
B −2
C 2
D 4

SOLUTION:
$$6x - 3y = 30 \rightarrow (1)$$
$$4x = 2 - y \rightarrow (2)$$

Solve (2) for y.

$$4x = 2 - y$$
$$4x - 2 = -y$$
$$y = -4x + 2$$

Substitute $y = -4x + 2$ in (1) and solve for x.

$$6x - 3(-4x + 2) = 30$$
$$6x + 12x - 6 = 30$$
$$18x = 36$$
$$x = 2$$

Substitute $x = 2$ in $y = -4x + 2$ and simplify.

$$y = -4(2) + 2$$
$$= -6$$

Thus, $x + y = -4$.

A is the correct answer.

ANSWER:
A

Solve each inequality. Check your solution.

71. $3^{n-2} > 27$

SOLUTION:
$$3^{n-2} > 27$$
$$3^{n-2} > 3^3$$
$$n - 2 > 3$$
$$n > 5$$

ANSWER:
$\{n \mid n > 5\}$

72. $2^{2n} \le \frac{1}{16}$

SOLUTION:

$$2^{2n} \le \frac{1}{16}$$
$$2^{2n} \le 2^{-4}$$
$$2n \le -4$$
$$n \le -2$$

ANSWER:
$\{n | n \le -2\}$

73. $16^{n} < 8^{n+1}$

SOLUTION:

$$16^{n} < 8^{n+1}$$
$$2^{4n} < 2^{3n+3}$$
$$4n < 3n + 3$$
$$n < 3$$

ANSWER:
$\{n | n < 3\}$

74. $32^{5p+2} \ge 16^{5p}$

SOLUTION:

$$32^{5p+2} \ge 16^{5p}$$
$$2^{25p+10} \ge 2^{20p}$$
$$25p + 10 \ge 20p$$
$$5p \ge -10$$
$$p \ge -2$$

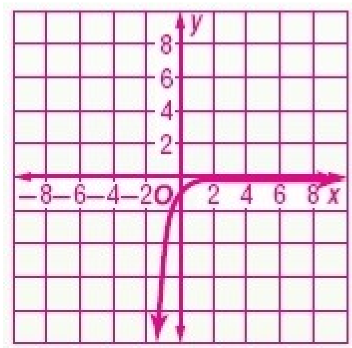
ANSWER:
$\{p | p \ge -2\}$

Graph each function.

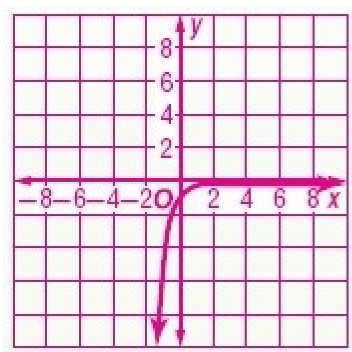
75. $y = -\left(\frac{1}{5}\right)^{x}$

SOLUTION:
Make a table of values. Then plot the points and sketch the graph.

x	y
–1	–5
0	–1
2	–0.04
4	–0.0016
6	–0.0001



ANSWER:

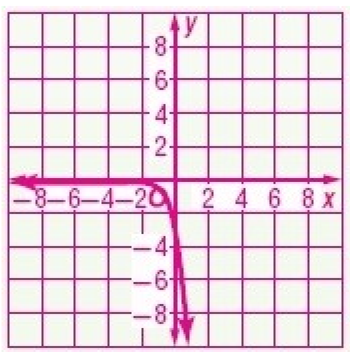


76. $y = -2.5(5)^x$

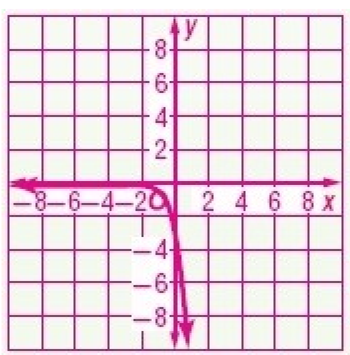
SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	y
−6	−0.002
−4	−0.004
−1	−0.5
0	−2.5
1	−12.5



ANSWER:

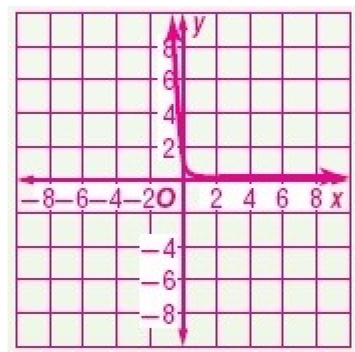


77. $y = 30^{-x}$

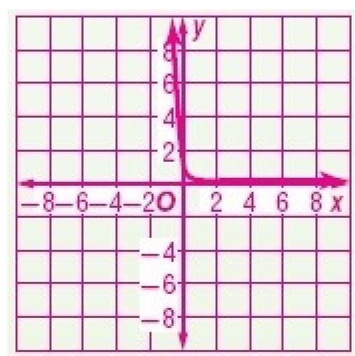
SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	y
−1	30
0	1
2	0.0011
4	0
6	0



ANSWER:

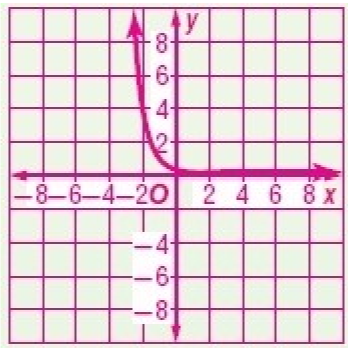


78. $y = 0.2(5)^{-x}$

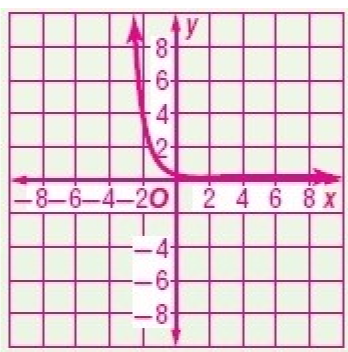
SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	y
–3	25
–2	5
0	0.2
2	0.0080
4	0.0003



ANSWER:



79. **GEOMETRY** The area of a triangle with sides of length a, b, and c is given by $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$. If the lengths of the sides of a triangle are 6, 9, and 12 feet, what is the area of the triangle expressed in radical form?

SOLUTION:

$$\begin{aligned} s &= \frac{1}{2}(a+b+c) \\ &= \frac{1}{2}(6+9+12) \\ &= \frac{27}{2} \end{aligned}$$

Area of the triangle:

$$\begin{aligned} \sqrt{s(s-a)(s-b)(s-c)} &= \sqrt{\frac{27}{2}\left(\frac{27}{2}-6\right)\left(\frac{27}{2}-9\right)\left(\frac{27}{2}-12\right)} \\ &= \sqrt{\frac{27}{2}\left(\frac{15}{2}\right)\left(\frac{9}{2}\right)\left(\frac{3}{2}\right)} \\ &= \frac{27}{4}\sqrt{15}\text{ ft}^2 \end{aligned}$$

ANSWER:

$\frac{27\sqrt{15}}{4}\text{ ft}^2$

80. **GEOMETRY** The volume of a rectangular box can be written as $6x^3 + 31x^2 + 53x + 30$ when the height is $x + 2$.

a. What are the width and length of the box?

b. Will the ratio of the dimensions of the box always be the same regardless of the value of x? Explain.

SOLUTION:

a.

Divide $6x^3 + 31x^2 + 53x + 30$ by $x + 2$.

$\underline{-2}\vert$	6	31	53	30
	0	–12	–38	–30
	6	19	15	$\vert$0

$$6x^3 + 31x^2 + 53x + 30 = (x+2)(6x^2 + 19 + 15)$$
$$= (x+2)(2x+3)(3x+5)$$

So, the width and length of the rectangular box are $2x + 3$ and $3x + 5$.

b.

No; for example, if $x = 1$, the ratio is 3:5:8, but if $x = 2$, the ratio is 4:7:11. The ratios are not equivalent.

ANSWER:

a. $2x + 3$ and $3x + 5$

b. No; for example, if $x = 1$, the ratio is 3:5:8, but if $x = 2$, the ratio is 4:7:11. The ratios are not equivalent.

81. **AUTO MECHANICS** Shandra is inventory manager for a local repair shop. She orders 6 batteries, 5 cases of spark plugs, and two dozen pairs of wiper blades and pays $830. She orders 3 batteries, 7 cases of spark plugs, and four dozen pairs of wiper blades and pays $820. The batteries are $22 less than twice the price of a dozen wiper blades. Use augmented matrices to determine what the cost of each item on her order is.

SOLUTION:

The augmented matrix that represents the situation is

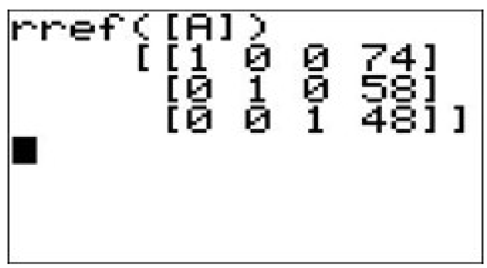
$$\left[\begin{array}{ccc|c} 6 & 5 & 2 & 830 \\ 3 & 7 & 4 & 820 \\ 1 & 0 & -2 & -22 \end{array}\right].$$

Use the graphing calculator to solve the system.

KEYSTROKES: 2ND [MATRIX] ► ► ENTER 3 ENTER 4 ENTER 6 ENTER 5 ENTER 2 ENTER 830 ENTER 3 ENTER 7 ENTER 4 ENTER 820 ENTER 1 ENTER 0 ENTER (–) 2 ENTER (–) 22 ENTER

Find the reduced row echelon form (rref).

KEYSTROKES: 2ND [QUIT] 2ND [MATRIX] ► ALPHA [B] 2ND [MATRIX] ENTER) ENTER



The first three columns are the same as a 3×3 identity matrix.

Thus, batteries cost $74, spark plugs costs $58 and wiper blades costs $48.

ANSWER:

batteries, $74; spark plugs, $58; wiper blades, $48

Solve each equation or inequality. Check your solution.

82. $9^x = \frac{1}{81}$

SOLUTION:

$$9^x = \frac{1}{81}$$
$$9^x = 9^{-2}$$
$$x = -2$$

ANSWER:

−2

83. $2^{6x} = 4^{5x + 2}$

SOLUTION:

$$2^{6x} = 4^{5x+2}$$
$$2^{6x} = 2^{10x+4}$$
$$6x = 10x + 4$$
$$-4x = 4$$
$$x = -1$$

ANSWER:

−1

84. $49^{3p + 1} = 7^{2p - 5}$

SOLUTION:

$$49^{3p+1} = 7^{2p-5}$$
$$7^{6p+2} = 7^{2p-5}$$
$$6p + 2 = 2p - 5$$
$$4p = -7$$
$$p = -\frac{7}{4}$$

ANSWER:

$-\frac{7}{4}$

85. $9^{x^2} \le 27^{x^2-2}$

SOLUTION:

$$9^{x^2} \le 27^{x^2-2}$$
$$3^{2x^2} \le 3^{3x^2-6}$$
$$2x^2 \le 3x^2 - 6$$
$$x^2 \le 6$$
$$x \le \pm\sqrt{6}$$

ANSWER:

$\left\{x \mid x \le -\sqrt{6} \text{ or } x \ge \sqrt{6}\right\}$